

# Color Optical Flow

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## Abstract

*Grayscale optical-flow methods have long been the focus of methods for recovering optical flow. Optical flow recovery from color-images can be implemented using direct methods, i.e. without using computationally costly iterations or search strategies. The quality of recovered optical flow can be assessed and tailored after processing, providing an effective, efficient tool for motion estimation.*

*In this paper, a brief introduction to optical flow is presented, the optical flow constraint equation and its extension to color images is presented. New methods for solving this extended equation are given. Results of applying these methods to two synthetic image sequences are presented.*

## 1 Introduction

Optical flow is a useful tool for many tasks in computer vision. It has been applied to problems of motion-segmentation, time-to-contact and three-dimensional reconstruction (structure from motion) among others. Traditionally, most researchers in this field have focussed their efforts on extending Horn and Shunck [8] or Lucas and Kanade's [9] methods, all working with grayscale intensity images.

Color image sequences have been largely ignored, despite three planes of information being available instead of one. Golland proposed and discussed two simple methods which incorporate color information [7]. She investigated the RGB, normalized RGB and HSV color models. Her results indicated that color methods provide a good estimate of the flow in image regions of non-constant color.

This paper compares traditional grayscale with Golland's methods and two new color methods. It also describes the proposed extension of grayscale methods into color.

## 2 Optical flow

The optical flow of an image sequence is a set of vector fields, relating each image to the next. Each vector field represents the apparent displacement of each pixel from image to image. If we assume the pixels conserve their intensity, we arrive at the "brightness conservation equation",

$$I(x, y, t) = I(x + dx, y + dy, t + dt) \quad (2.1)$$

where  $I$  is an image sequence,  $[dx, dy]$  is the displacement vector for the pixel at coordinate  $[x, y]$  and  $t$  and  $dt$  are the frame and temporal displacement of the image sequence. The idea of brightness conservation and optical flow were first proposed by Fennema [6].

The obvious solution to 2.1 is to use template-based search strategies. A template of a certain size around each pixel is created and the best match is searched for in the next image. The best match is usually found using correlation, sum of absolute difference or sum of squared difference metrics. This process is often referred to as block-matching. Such a search strategy is computationally costly and generally doesn't represent sub-pixel displacements.

Most methods presented in the last twenty years have been gradient-based. They solve the differential form of 2.1, derived by Taylor expansion. After discarding higher order terms, this is

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0 \quad (2.2)$$

Here we have two unknowns in only one equation, the problem is ill-posed and extra constraints must be imposed in order to arrive at a solution.

The two most commonly used and earliest optical flow recovery methods in this category are briefly outlined below, Horn and Shunck's [8] and Lucas and Kanade's [9] optical flow methods. These and other traditional methods are outlined and quantitatively compared in Barron et.al. [4][3].

## 2.1 Horn and Schunck

Horn and Shunck [8] were the first to impose a global smoothness constraint, assuming the flow to be smooth across the image. Their minimization function,

$$\int \int (I_x u + I_y v + I_t)^2 + \lambda^2 (\|\nabla u\|_2^2 + \|\nabla v\|_2^2) dx dy \quad (2.3)$$

can be expressed as a pair of Gauss-Siedel iterative equations,

$$u_{n+1} = u_n - \frac{I_x [I_x u_n + I_y v_n + I_t]}{\alpha^2 + I_x^2 + I_y^2} \quad (2.4)$$

and

$$v_{n+1} = v_n - \frac{I_y [I_x u_n + I_y v_n + I_t]}{\alpha^2 + I_x^2 + I_y^2} \quad (2.5)$$

## 2.2 Lucas and Kanade

Lucas and Kanade [9] put forth the assumption of constant flow in a local neighborhood. Their method is generally implemented with neighborhoods of size  $5 \times 5$  pixels centered around the pixel whose displacement is being estimated. Measurements nearer the centre of the neighborhood are given greater weight in the weighted-least-squares formulation.

## 2.3 Other methods

Later methods generally extended of these two traditional methods. More recently, researches have been focusing on using concepts of robustness to modify Lucas and Kanade's method [2][1]. These methods choose a function other than the squared difference of the measurement to the line of fit (implicit in least squares calculation) to provide an estimate of the measurement's contribution to the best line. Functions are chosen so that outliers are ascribed less weight than those points which lie close to the line of best fit. This formulation results in a method which utilises iterative numerical methods, e.g. gradient descent or successive over-relaxation.

## 3. Using color images

Recovering optical flow from color images seems to have been long overlooked by researchers in the field of image processing and computer vision. Ohta [11] mentioned the idea, but presented no algorithms or methods. Golland proposed some methods in a thesis and a related paper [7]. She proposed using the three color planes to infer three equations, then solving these using standard least squares tech-

niques.

$$\begin{aligned} \frac{\partial I_R}{\partial x} u + \frac{\partial I_R}{\partial y} v + \frac{\partial I_R}{\partial t} &= 0 \\ \frac{\partial I_G}{\partial x} u + \frac{\partial I_G}{\partial y} v + \frac{\partial I_G}{\partial t} &= 0 \\ \frac{\partial I_B}{\partial x} u + \frac{\partial I_B}{\partial y} v + \frac{\partial I_B}{\partial t} &= 0 \end{aligned} \quad (3.1)$$

The other idea proposed by Golland was the concept of "color conservation". By constructing a linear system to solve from only color components, e.g. Hue and Saturation from the HSV color model, the illumination is allowed to change, the assumption is now that the color, rather than brightness is conserved.

### 3.1. Color Models

Three color models have been implemented and tested in this paper. These are RGB, HSV and normalized RGB.

The RGB (Red, Green, Blue) color model decomposes colors into their respective red, green and blue components.

Normalized RGB is calculated as

$$N = R + G + B, \quad R_n = \frac{R}{N}, \quad G_n = \frac{G}{N}, \quad B_n = \frac{B}{N} \quad (3.2)$$

each color being normalized by the sum of all colors at that point. If the color value at that point is zero the normalized color at that point is taken as zero.

The HSV (Hue, Saturation, Value) model expresses the intensity of the image (V) independently of the color (H, S). Optical flow based purely on V is relying on brightness conservation. Conversely, methods which are based on H and S rely purely on color conservation. Methods which combine the two incorporate both assumptions.

Similar to HSV, the YUV model decomposes the color as a brightness (Y) and a color coordinate system (U,V). The difference between the two is the description of the color plane. H and S describe a vector in polar form, representing the angular and magnitudinal components respectively. Y, U and V, however, form an orthogonal euclidean space.

An alternative to these spaces is *CIE perceptually linear color space* also known as UCS (Uniform Chromaticity Scale). This color system has the advantage of euclidean distances in color space corresponding linearly to perception of color or intensity change.

Neither YUV, nor UCS have been implemented, though this is the next step in analysing color optical flow.

### 3.2. Methods

Two obvious methods for arriving at a solution to the extended brightness conservation equation 3.2 are apparent:

- Disregarding one plane so as to solve quickly and directly, using Gaussian Elimination.
- Solving the overdetermined system as is, using either least squares or pseudo-inverse methods.

Disregarding one of the planes arbitrarily may throw away data that is more useful to the computation of optical flow than those kept. However, if speed of the algorithm is of the essence, disregarding one plane reduces memory requirements and computational cost. Another possibility is merging two planes and using this as the second equation in the system. Numerical stability of the solution should be considered when constructing each system. By using the simple method of pivoting it is possible to ensure the best possible conditioning of the solution.

The methods of least squares and pseudo-inverse calculation are discussed in nearly all linear algebra texts.

A simple neighborhood least-squares algorithm, akin to Lucas and Kanade’s [9], though not utilising weighting, has also been implemented. Values in a  $3 \times 3 \times 3$  neighborhood around the center pixel were incorporated into a large, overdetermined system.

Another option for the computation of optical flow from color images is to estimate the optical flow of each plane using traditional grayscale techniques and then fuse these results to recover one vector field. This fusion has been implemented here by simply selecting the estimated vector with the smallest intrinsic error at each point.

All of the the methods mentioned above have been implemented and compared in this study.

## 4 Error Analysis

Image reconstruction is a standard technique for assessing the accuracy of optical flow methods, especially for sequences with unknown ground truth (see Barron and Lin [5]). The flow field recovered from an optical flow method is used to warp the first image into a reconstructed image, an approximation to the second image. If the optical flow is accurate then the reconstructed image should be the same as the second image in the image sequence. Generally, the RMS error of the entire reconstructed image is taken as the image reconstruction error. However, it is advantageous to calculate the image reconstruction error at each point in the image. This enables a level of thresholding in addition to, or instead of culling estimates with high intrinsic error.

The density of the flow field after thresholding at chosen image reconstruction errors can also be used to compare different methods. This is the method applied for comparison herein.

Method	$64 \times 64$	$128 \times 128$	$240 \times 320$
Color (Least Sq)	0.66	2.11	9.24
Color (G E, 3 rows)	0.05	0.24	1.58
Color (GE, pivot, 2 rows)	0.03	0.14	0.54
H & S (20 its)	0.23	0.45	2.18
Lucas & Kanade	0.73	3.52	15.19
Nagel	1.63	6.70	36.68
Uras, et. al.	2.06	8.19	37.07
NCC	3.00	9.08	40.20
Black & Anandan	0.60	2.94	19.08

**Table 1. Time taken for computation**

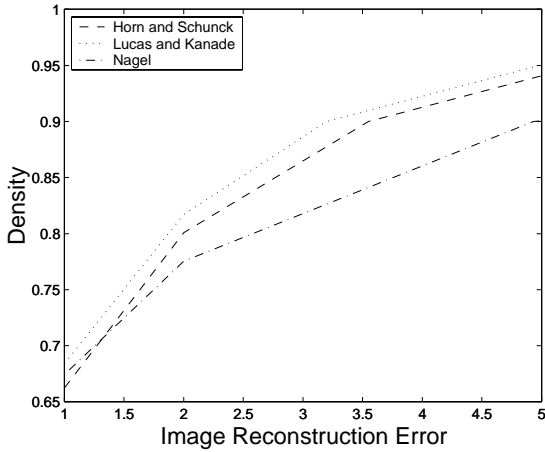
## 5. Results and Discussion

Table 1 compares the time taken for recovery of optical flow using Matlab<sup>®</sup>, excluding low-pass filtering and derivative calculation times. The times recorded from computation on a 700Mhz Pentium III<sup>®</sup> processor. This highlights the drastic decrease in computational cost of direct color methods. The two row partial pivoting Gaussian Elimination method is seen to perform at approximately 20Hz. Compared to Horn and Shunck’s method [8], the best performer in the field of grayscale methods, this represents an approximately fourfold increase in speed.

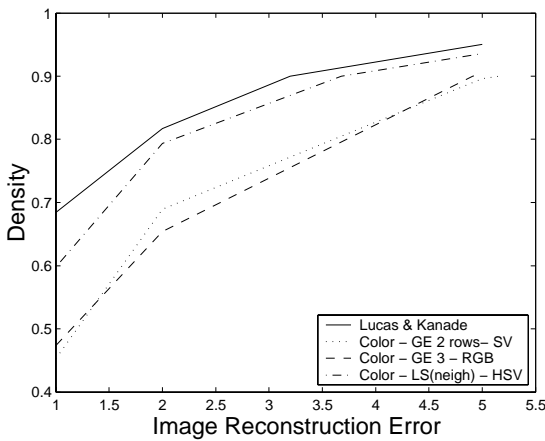
Figure 5.1 compares three common grayscale optical flow methods; Horn and Shunck [8], Lucas and Kanade [9] and Nagel [10]. This figure illustrates the density of the computed flow field when thresholded at chosen image reconstruction errors. It is seen that Lucas and Kanade’s method [9] slightly outperforms Horn and Shunck’s [8] method, which itself performs better than Nagel’s [10] method at image reconstruction errors  $> \approx 1.35$ .

Figure 5.2 compares the performance of Lucas and Kanade’s [9] with three color methods. The first frame of this image sequence is shown in figure 5.3. This sequence was translating with velocity [-1,-1] pixels per frame. The three color methods shown here are gaussian elimination (with pivoting) of the saturation and value planes of HSV, Gaussian elimination of RGB color planes and neighborhood least squares. Neighborhood least squares is seen to perform the best out of the color methods, closely approximating Lucas and Kanade at higher densities. Both gaussian elimination versions performed worse than the others.

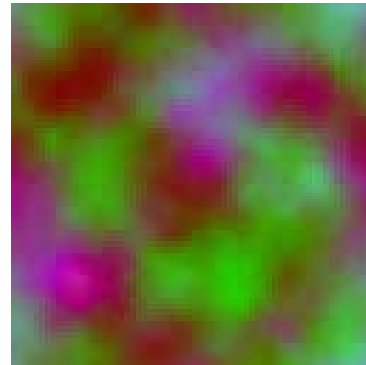
An image sequence displaying a one degree anticlockwise rotation around the center of the image was used to assess three other color optical flow methods. Pixel displacement ranges between zero and 1.5 pixels per frame. The methods compared were “Color Constancy” [7], least squares solution to 3.2 [7] and Combined-Horn and Shunck. Horn and Shunck’s [8] (grayscale) algorithm was used as a yardstick for this comparison. The results are displayed in



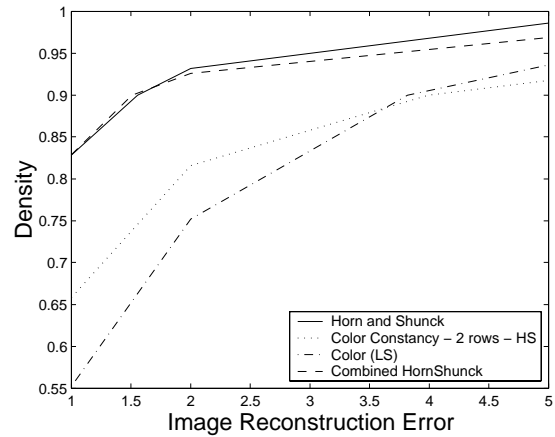
**Figure 5.1. Comparison of Grayscale methods applied to translating colored clouds**



**Figure 5.2. Comparison of gray and color methods applied to translating colored clouds**



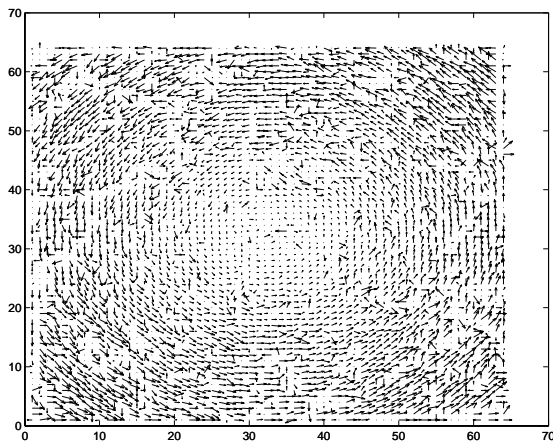
**Figure 5.3. First frame of the translating RGB clouds sequence**



**Figure 5.4. Comparison of techniques applied to a rotating image sequence**

figure 5.4. Combined-Horn and Shunck applied Horn and Shunck optical flow recovery to each plane of the RGB image and fused them into one flow field utilising a winner-takes-all strategy based on their associated error. It can be seen that the Combined-Horn and Shunck method performed similarly to Horn and Shunck [8]. The methods of least squares [7] and direct solution of the color constancy equation [7] did not perform as well.

Figure 5.5 gives an example of the optical flow recovered by the neighborhood least squares algorithm. This corresponds to the rotating image sequence. Larger vectors (magnitude greater than 5) have been removed and replaced with zero vectors. This field has a density of 95%.



**Figure 5.5. Optical flow recovered by direct two-row optical flow and thresholding**

## 6 Conclusion and future work

Color optical flow has been shown to be quite simple to compute and to have a level of accuracy similar to traditional grayscale methods. The speed of these algorithms is a significant benefit; the linear optical flow methods presented run substantially faster than grayscale, non-linear methods.

YUV and UCS color models will be implemented and compared.

Accuracy of the neighborhood least squares approach can be improved in a number of ways. Using robust methods, e.g. least-median of squares [2], could provide a much better estimate of the correct flow. Applying the weighted least squares approach of Lucas and Kanade [9] could likewise improve the results.

A better data-fusion algorithm could be used to improve the Combined-Horn and Shunck method. The three flows being combined could be calculated using any grayscale method.

Methods that iterate towards a solution usually perform better with a good initial starting estimate. Color-optical flow could be used to provide this estimate, speeding the computation of some of the slower, well-known grayscale methods.

These issues will be investigated in future work.

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