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Separating a Colour Texture from an Arbitrary Background

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Abstract. Partially supervised segmentation, that is, segmentation with always incomplete training data has many practical applications in image analysis and retrieval. This paper proposes a new algorithm for finding regions of a single texture in an arbitrary colour image. The texture is specified by a given training sample. The algorithm exploits colour space vector quantization, colour thresholding, and similarity between characteristic grey level cooccurrence histograms over a moving window around each pixel and over the whole training sample. Experiments show this algorithm effectively finds various homogeneous textures in complex backgrounds.

1 Introduction

Image segmentation has many practical applications for identifying regions of interest in remote sensing of the Earth's surface, medical diagnostics, and industrial vision. Many of the known models of unsupervised and supervised texture segmentation belong to two basic groups: statistical models assuming that similarity of textures is represented in terms of sufficient signal statistics collected over the whole regions (e.g. [4, 10, 13]) and feature-based schemes where similar textures are described by similar local feature vectors (e.g. [11, 12, 17, 22, 23]). Several known algorithms of colour image segmentation (e.g. [2, 6, 16]) exploit only the pixel-wise colour information. Today, the combined colour and texture segmentation (e.g. [1, 5, 8, 15]) is of more interest. Moreover, in many practical cases, only the regions with specific known textures have to be found.

The problem of separating a single known texture from an arbitrary background is of considerable practical value. For example, a typical application is the content-based image retrieval (see, e.g., comprehensive surveys in [3, 19, 20]) in a large database given a small patch of example. Such a partially supervised segmentation cannot be performed using the existing supervised or unsupervised methods that pursue the goal of finding all the regions of different spatially homogeneous textures in each image. The supervised techniques assume that all the training samples of such textures are available, while the unsupervised methods rely on no training data at all. But in the both cases the images have to be split onto the set of homogeneous textured regions, and only a few of these latter are of practical interest. This paper investigates a new algorithm that separates the regions similar to a known homogeneous colour texture (the training sample) from an arbitrary and mostly inhomogeneous background.

2 Segmentation Algorithm: Basic Steps

The proposed algorithm performs colour space vector quantization (CSVQ) to reduce the data volume. Then colour pixel-wise thresholding excludes the pixels with notably different colours with respect to the training sample, and the remaining candidate areas are converted into the greyscale image. A generic Gibbs random field model (GGRF) of spatially homogeneous textures [10] is used for selecting a characteristic subset of grey level cooccurrence histograms (GLCH) to represent the training sample. The desired texture region is obtained by thresholding the total distances between the normalized GLCHs in a moving window around each pixel of the image to be segmented and the like global GLCHs over the whole training sample. The basic steps of the algorithm are as follows:

1. Create a codebook with a fixed number of codevectors for the training sample using CSVQ.
2. Find a probabilistic colour threshold using a Gaussian mixture (GM) approximation of the codebook.
3. Select the candidate areas in the original image using the above threshold.
4. Convert the training sample and the above candidate area image into the greyscale images.
5. Find a characteristic subset of the normalized GLCHs.
6. Find an empirical distribution of distances between the local normalized GLCHs in a fixed moving window around each pixel and the like global GLCHs over the whole greyscale training sample.
7. Select a distance threshold using the above distribution.
8. Find the desired region map by thresholding the distances between the local normalized GLCHs in a fixed moving window around each pixel over the greyscale image of the candidate areas and the global GLCHs for the training sample.

Colour aerial image in Fig. 1 provided by the Institute of Communication Theory and Signal Processing (TNT), University of Hannover, is used below as a test example to search for each of the four training regions: field (top left image in Fig. 2), vegetation (top left image in Fig. 3), residential area (top left image in Fig. 4), and industrial area (top left image in Fig. 5).

CSVQ and Colour Tresholding. The CSVQ of a training sample $\mathbf{S}^{tr}$ is based on the LBG algorithm [14] and produces a codebook $\mathbf{B}$ with a fixed number N of codevectors where N is less than the total number of different colours in $\mathbf{S}^{tr}$.

Let $\mathbf{S} = [\mathbf{s}_i = [s_{i,\mathrm{r}}, s_{i,\mathrm{g}}, s_{i,\mathrm{b}}] : i = 1, \ldots, M; \mathbf{s}_i \in \mathbf{Q}^3]$ denote a digital colour image. Here, $\mathbf{Q} = \{0, \ldots, Q\}$ is a finite integer set and $\mathbf{s}_i$ is a vector in the 3D RGB colour space for the image position i ($s_{i,r} \in \mathbf{Q}$, $s_{i,g} \in \mathbf{Q}$, and $s_{i,b} \in \mathbf{Q}$).

Fig. 1. Original colour aerial image of the Earth's surface provided by the Institute of Communication Theory and Signal Processing (TNT), University of Hannover, Hannover, Germany.

The position i is a shorthand notation of the 2D integer Cartesian coordinates $i = (x, y)$.

Let $\mathbf{B} = [\mathbf{b}_k : k = 1, \ldots, N]$ represent a codebook with a pre-defined number N of codevectors $\mathbf{b}_k = [b_{k,r}, b_{k,g}, b_{k,b}]$. Let $\mathbf{\Omega} = [\Omega_k : k = 1, \ldots, N]$ denote such a partition of the RGB colour space where each region Ω_k is associated with the codevector $\mathbf{b}_k$.

The CSVQ problem is stated as follows: given a training sample $\mathbf{S}^{tr} = [\mathbf{s}_j^{tr} : j = 1, \ldots, M^{tr}; \mathbf{s}_j^{tr} \in \mathbf{Q}^3]$ and a fixed number N of codevectors, find the codebook $\mathbf{B}$ and the patition $\mathbf{\Omega}$ resulting in the minimum average distortion Δ of the coded image with respect to the original one. It satisfies the nearest neighbour condition (the region Ω_k consists of all the colour vectors $\mathbf{s}_j^{tr}$ which are closer to the codevector $\mathbf{b}_k$ than to any other codevector) and the centroid condition (the codevector $\mathbf{b}_k$ is the average of all the colour vectors $\mathbf{s}_j^{tr}$ which are in the region Ω_k, and at least one training vector belongs to each region). The quantization replaces each colour vector in the image with the closest codevector, and the average distortion is given by the average squared error of the quantization. The CSVQ algorithm is as follows:

1. Given: a training sample $\mathbf{S}^{tr}$, a fixed small threshold $\epsilon > 0$, and an interpolation factor $0 < \tau < 1$
2. Set $N = 1$, and $\Omega_1 = \{\mathbf{s}_j^{tr} : j = 1, \ldots, M^{tr}\}$, and calculate:
$$\mathbf{b}_1^* = \frac{\sum_{j=1}^{M^{tr}} \mathbf{s}_j^{tr}}{M^{tr}};\ \ \tilde{\mathbf{s}}_j^{tr} = \mathbf{b}_1^*, j = 1, \ldots, M^{tr}$$
$$\mathbf{v}(\Omega_1) = \mathbf{s}_j^{tr} : \max\{||\mathbf{s}_j^{tr} - \mathbf{b}_1^*||^2 : \forall \mathbf{s}_j^{tr} \in \Omega_1\};$$
$$\Delta^* = \frac{1}{3M^{tr}} \sum_{j=1}^{M^{tr}} ||\mathbf{s}_j^{tr} - \mathbf{b}_1^*||^2$$
3. Split the codevectors: for $k = 1, \ldots, N$, set
$$\mathbf{b}_k^{(0)} = \mathbf{b}_k^* + \tau(\mathbf{v}(\Omega_k) - \mathbf{b}_k^*);\ \ \mathbf{b}_{N+k}^{(0)} = \mathbf{b}_k^* - \tau(\mathbf{v}(\Omega_k) - \mathbf{b}_k^*);\ \ N = 2N$$

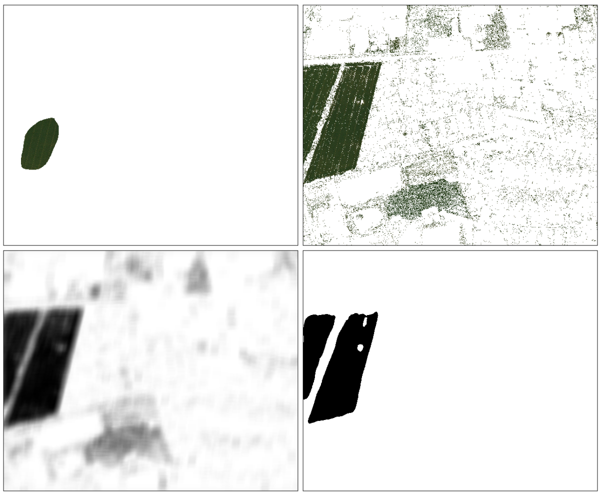

Fig. 2. Top left: the training region "field"; top right: the candidate regions; bottom left: the grey-coded distance map; bottom right: the final region map.

4. Iterative updating: set the iteration index $t = 0$ and $\Delta^{(0)} = \Delta^*$, then:
   (a) for $j = 1, \ldots, M^{tr}$, $\widetilde{\mathbf{s}}_j^{tr} = \mathbf{b}_k^{(t)} : \min\{||\mathbf{s}_j^{tr} - \mathbf{b}_k^{(t)}||^2 : k = 1, \ldots, N\}$
   (b) for $k = 1, \ldots, N$,
   - update $\Omega_k = \{\mathbf{s}_j^{tr} : \widetilde{\mathbf{s}}_j^{tr} = \mathbf{b}_k^{(t)}, j = 1, \ldots, M^{tr}\}$
   - update the codevector: $\mathbf{b}_k^{(t+1)} = \dfrac{\sum_{\widetilde{\mathbf{s}}_j^{tr} = \mathbf{b}_k^{(t)}} \mathbf{s}_j^{tr}}{\sum_{\widetilde{\mathbf{s}}_j^{tr} = \mathbf{b}_k^{(t)}} 1}$
   - update $\mathbf{v}(\Omega_k) = \mathbf{s}_j^{tr} : \max\{||\mathbf{s}_j^{tr} - \mathbf{b}_k^{(t+1)}||^2 : \forall \mathbf{s}_j^{tr} \in \Omega_k\}$

   (c) set $t = t + 1$
   (d) $\Delta^t = \frac{1}{3M^{tr}} \sum_{j=1}^{M^{tr}} ||\mathbf{s}_j^{tr} - \widetilde{\mathbf{s}}_j^{tr}||^2$
   (e) if $\frac{(\Delta^{t-1} - \Delta^t)}{\Delta^{t-1}} > \epsilon$, go back to Step 4(a)
   (f) set $\Delta^* = \Delta^t$. For $k = 1, \ldots, N$, set $\mathbf{b}_k^* = \mathbf{b}_k^{(t)}$
5. Repeat Steps 3 and 4 until the desired number of codevectors is obtained.

Pixels in the image $\mathbf{S}$ to be segmented having colours that differ much from the ones of the training sample $\mathbf{S}^{tr}$ can be excluded from the separation process. To perform such a colour probabilistic thresholding, the codebook $\mathbf{B} = [\mathbf{b}_k : k = 1, \ldots, N]$ obtained from the training sample $\mathbf{S}^{tr}$ is approximated by a mixture of Gaussian probability distributions (GM). The GM has L Gaussian components $\mathcal{N}(\mathbf{s}|\mathbf{m}_l, \mathbf{\Sigma}_l)$ specified each by the mean colour $\mathbf{m}_l$ and the covariance matrix $\mathbf{\Sigma}_l$ and mixed together with the prior probabilities α_l, $l = 1, \ldots, L$:

$$p(\mathbf{b}_k|\mathbf{\Lambda}_L) = \sum_{l=1}^{L} \alpha_l \, \mathcal{N}(\mathbf{b}_k|\mathbf{m}_l, \mathbf{\Sigma}_l) \ . \tag{1}$$

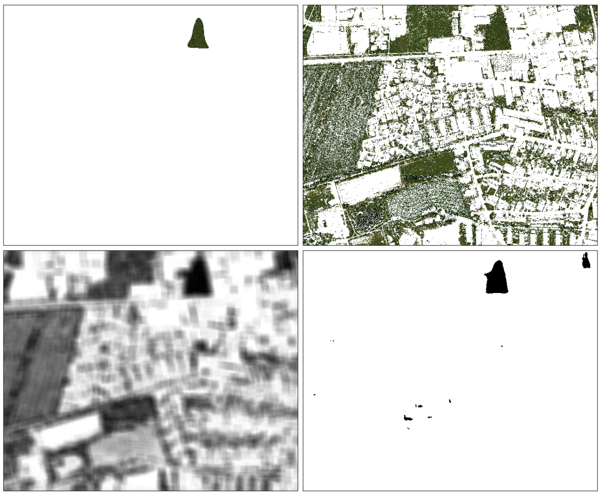

Fig. 3. Top left: the training region "vegetation"; top right: the candidate regions; bottom left: the grey-coded distance map; bottom right: the final region map.

where $\mathbf{\Lambda}_L = \{\alpha_l, \mathbf{m}_l, \Sigma_l : l = 1, \ldots, L\}$.

The expectation-maximization (EM) algorithm ([7, 18, 24]) is usually used for finding the maximum likelihood estimate $\widetilde{\Lambda}_L$ of the GM parameters Λ_L for the codebook $\mathbf{B}$. This algorithm has three main problems: (i) convergence to the local maximum with a possibility to converge to a boundary point of the parameter space with the unbounded likelihood, (ii) critical dependence on initialization, and (iii) estimation of the number of the mixture components. There exist a few approaches of how to resolve these problems. One of the best solutions is provided by the agglomerative EM (AEM) algorithm with the mixture minimum description length (MMDL) criterion [9].

The candidate areas for further segmentation are selected by thresholding the probabilities $\mathbf{Pr}(\mathbf{S}|\widetilde{\Lambda}_L) = [p(\mathbf{s}_i|\widetilde{\Lambda}_L) : i = 1, \ldots, M]$ of colours in each pixel of the original image $\mathbf{S}$ to be segmented. Figure 6 presents relative frequency distributions (in terms of their negative logarithmic values) of the quantised probabilities of colours for all the pixels in the original image $\mathbf{S}$ and the training samples $\mathbf{S}^{tr}$ of the field and residential area presented in Figs. 2 and 4. In these examples the codebook $\mathbf{B}$ for each training sample contains 64 codevectors. The distributions presented show that only a small part of the colours in the original image are similar to the training field, while much more colours are similar to the training residential area. The threshold ξ_f for the training colours eliminates the "noisy" colours with probabilities smaller than f.

Images $\mathbf{S}' = [\mathbf{s}'_i : i = 1, \ldots, M]$ of the candidate regions are formed by thresholding the original colour image $\mathbf{S}$ as follows:

$\mathbf{s}'_i = \mathbf{s}_i$, if $p(\mathbf{s}_i|\widetilde{\Lambda}_L) \geq \xi_f$;
$\mathbf{s}'_i = [Q, Q, Q]$, otherwise (the white background).

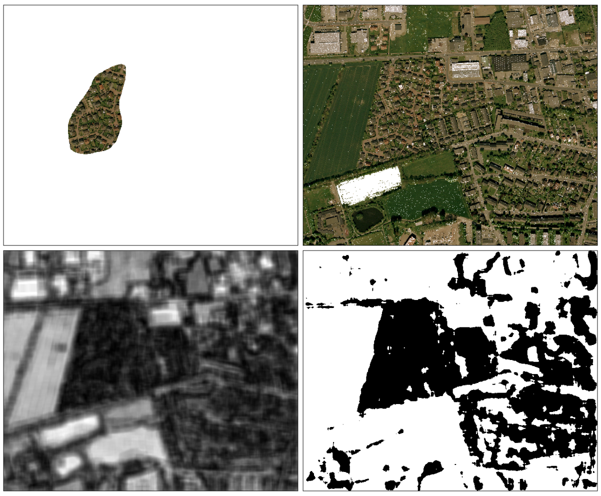

Fig. 4. Top left: the training region "residential area"; top right: the candidate regions; bottom left: the grey-coded distance map; bottom right: the final region map.

The top right pictures in Figs. 2 - 5 demonstrate the candidate region images $\mathbf{S}'$ after applying the threshold $\xi_{0.005}$ to the original colour image $\mathbf{S}$, given 64 codevectors in the codebook $\mathbf{B}$ of the training samples $\mathbf{S}^{tr}$ "field", "vegetation", "residential area", and "industrial area", respectively.

Although such a probabilistic thresholding separates colours similar to the training sample from the background, the separation is incomplete and different candidate textures still have to be discriminated.

Texture Similarity Measure. Spatially homogeneous image textures can be modelled as samples of a generic Gibbs random field (GGRF) with multiple pairwise pixel interactions [10]. Let $\mathbf{G} = [g_i : i = 1, \ldots, M;\, g_i \in \mathbf{Q}]$ and $\mathbf{G}^{tr} = [g^{tr}_j : j = 1, \ldots, M^{tr};\, g^{tr}_j \in \mathbf{Q}]$ denote the greyscale versions of the candidate colour regions $\mathbf{S}'$ and the training sample $\mathbf{S}^{tr}$, respectively.

Characteristic geometric structure of interactions and Gibbs potentials giving quantitative interaction strengths for a particular texture are estimated from the training sample of the texture. The estimation yields a characteristic subset of pixel neighbours $\mathbf{A}$ specifying most "energetic" translation invariant families of interacting pixel pairs $\mathbf{C}_a = \{(i, i+a) : i \in \{1, \ldots, M\}; i + a \in \{1, \ldots, M\}\}$; $a \in \mathbf{A}$. Each family is presented in the GGRF model by the corresponding GLCH acting as a sufficient statistic.

Let $\mathbf{F}_a(\mathbf{G}^{tr}) = [F_a(q,\, s|\mathbf{G}^{tr}) : q,\, s \in \mathbf{Q}]$ and $\mathbf{F}_{a,i}(\mathbf{G}) = [F_{a,i}(q, s|\mathbf{G}) : q,\, s \in \mathbf{Q}]$ denote the global normalized GLCH for the family $\mathbf{C}_a$ over the training sample $\mathbf{G}^{tr}$ and the like local normalized GLCH over the moving window $\widetilde{\mathbf{W}}$ around a position i in the image $\mathbf{G}$, respectively. Experiments with different

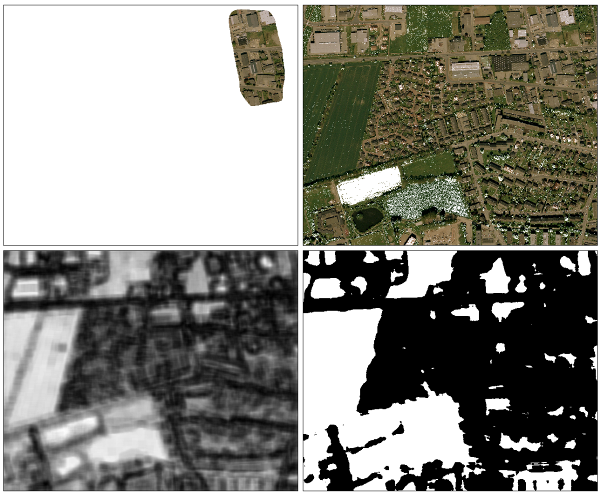

Fig. 5. Top left: the training region "industrial area"; top right: the candidate colour regions; bottom left: the grey-coded distance map; bottom right: the region map.

textures show that the symmetric χ^2-distance between these two GLCHs:

$$D_{a,i}(\mathbf{F}_{a,i}(\mathbf{G}), \mathbf{F}_a(\mathbf{G}^{tr})) = \sum_{q,\, s \in \mathbf{Q}} \frac{(F_{a,i}(q,s|\mathbf{G}) - F_a(q,s|\mathbf{G}^{tr}))^2}{F_{a,i}(q,s|\mathbf{G}) + F_a(q,s|\mathbf{G}^{tr})} \ .$$

has much less scatter for the training sample than, for instance, the pixel-wise Gibbs energies or conditional probabilities of signals. Therefore, for the $|\mathbf{A}|$ characteristic families, the similarity measure between $\mathbf{G}$ and $\mathbf{G}^{tr}$ can be defined as follows:

$$D_i(\mathbf{F}_i(\mathbf{G}), \mathbf{F}(\mathbf{G}^{tr})) = \frac{1}{|\mathbf{A}|} \sum_{a \in \mathbf{A}} D_{a,i}(\mathbf{F}_{a,i}(\mathbf{G}), \mathbf{F}_a(\mathbf{G}^{tr})) \ .$$

3 Experimental Results

In the experiments below, we used a 17×17 moving window, the reduced number $|\mathbf{Q}| = 16$ of grey levels, and only one ($|\mathbf{A}| = 1$) most energetic family of pixel pairs per each training sample $\mathbf{G}^{tr}$. Figures 7 and 8 show empirical distributions of the quantized χ^2-distances $D_i(\mathbf{F}_i(\mathbf{G}^{tr}), \mathbf{F}(\mathbf{G}^{tr}))$ over the training sample $\mathbf{G}^{tr}$ and $D_i(\mathbf{F}_i(\mathbf{G}), \mathbf{F}(\mathbf{G}^{tr}))$ over the image $\mathbf{G}$ to be segmented, the training samples $\mathbf{G}^{tr}$ being "field", "vegetation", "residential area", and "industrial area", respectively.

The main part of the distances in Fig. 7 (apart from 5% of the top-rank ones) are spread over the narrow ranges: e.g., the range $0.22 - 0.61$ ("field") and $0.08 - 0.30$ ("vegetation"). These ranges become wider in Fig. 8: $0.23 - 0.51$ ("residential area") and $0.25 - 0.81$ ("industrial area"). The separation threshold for the distances can be obtained from the corresponding training range.

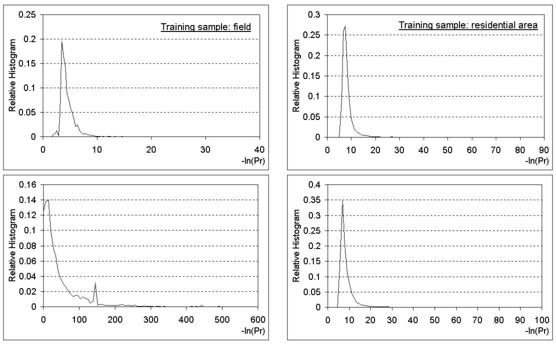

Fig. 6. Relative frequency distributions of the quantised probabilities of colours for the training sample (top row) and the original image (bottom row) with 64 codevectors in the codebook of the training samples “field” (left column) and “residential area” (right column).

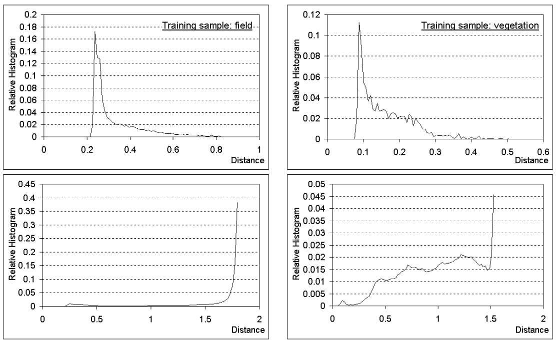

Fig. 7. Distibutions of the χ^2-distances: $D_i(\mathbf{F}_i(\mathbf{G}^{tr}), \mathbf{F}(\mathbf{G}^{tr}))$ (top) and $D_i(\mathbf{F}_i(\mathbf{G}), \mathbf{F}(\mathbf{G}^{tr}))$ (bottom) for the training samples $\mathbf{G}^{tr}$: “field” (left) and “vegetation” (right).

The bottom left pictures in Figs. 2-5 are the grey-coded distance maps for the candidate colour regions of the original colour image with the training samples $\mathbf{G}^{tr}$ “field”, “vegetation”, “residential area”, and “industrial area”, respectively; the darker the point, the smaller the distance. The bottom right pictures demonstrate the corresponding region maps (the dark areas indicate the desired regions). The separation is performed by thresholding the pixel-wise distances with the threshold that rejects 5% of the topmost training distances. Thus the proposed combination of colour and GLCH-based distance thresholding effectively separates such homogeneous textures as “field” and “vegetation” from their background. Similar but less accurate separation is obtained for the weakly homogeneous “residential area”. But the inhomogeneous “industrial area” yields a very low accuracy of separation because the obtained region covers both the residential and industrial portions of the image.

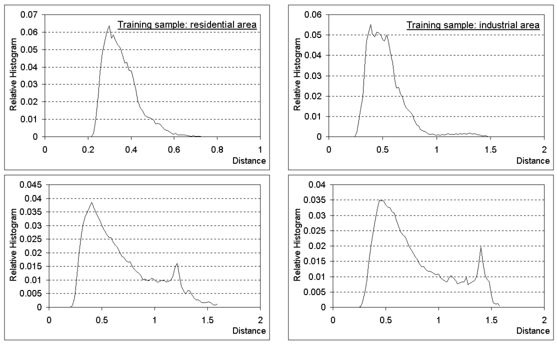

Fig. 8. Distributions of the χ^2-distances: $D_i(\mathbf{F}_i(\mathbf{G}^{tr}), \mathbf{F}(\mathbf{G}^{tr}))$ (top) and $D_i(\mathbf{F}_i(\mathbf{G}), \mathbf{F}(\mathbf{G}^{tr}))$ (bottom) for the training samples $\mathbf{G}^{tr}$: "residential area" (left) and "industrial area" (right).

4 Conclusions

These and other experiments show that the proposed technique results in quite accurate separation of a translation invariant texture from an arbitrary background. But this technique is inadequate for translation variant textures, and how to separate such textures is still an open question. As regarding the translation invariant stochastic textures and regular mosaics, it will be necessary in future to find out how to select more specific pixel neighbourhoods for separation and develop more efficient measures of similarity between the signal cooccurrence distreibutions collected over small windows.

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