Nonlinear processing techniques for P-wave Detection and Classification: A review of current methods and applications

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Abstract

In this paper we focus on the particular problem of extraction and classification of the P wave and the current techniques, both linear and nonlinear, which are employed. A brief description of the techniques is given along with specific examples of their application to the identification of Electrocardiographic P waves. Finally an analytical comparison of the sensitivity for each of these techniques is presented along with a discussion on their effectiveness and limitations.

1 Introduction

Heart disease is a major cause of death amongst Australians and New Zealanders and in 1999 caused almost 40% of all deaths in Australia and New Zealand [1][2]. Therefore research and development of technologies in the field of cardiology are of great interest and worth. In recent years many innovations in cardiac care have taken place, which have increased the tool base of modern cardiology. This has assisted the cardiologist in making more informed diagnosis thereby leading to prevention of serious conditions or the prescription of more beneficial treatment. Linear processing of biomedical signals have been implemented to great effect in the past [3,4,5], however current linear processing techniques are beginning to show limitations in their applications to the processing and analysis of biomedical signals. The limitations of linear processing techniques, such as the inability to discriminate between two signals of similar frequency content but difference phase information [6] can be overcome using relatively uncomplicated nonlinear methods of processing and analysis such as, nonlinear dynamics and representation, neural network analysis and non-linear filtering. In this paper several methods used to identify the p-wave are briefly described and their application to p-wave identification is then reviewed and a comparison of their effectiveness is presented. Finally a discussion of the relative merits and limitations of each method is offered.

1.1 The Electrocardiogram

The Electrocardiogram (ECG) is defined as the recordings made by the electrocardiograph, depicting the electrical activity of the heart [7]. Each segment or interval in the ECG signal (P-wave, QRS complex, ST segment) represents the electrical activity present in a specific part of the heartbeat cycle. The initial pulse formation in each section of the heart is conducted through the respective sections e.g. the Sinoatrial node provides the initial charge to the atria and the Atrioventricular node provides the initial charge to the ventricular mass, which comprises a number of separate sections. Arrhythmias arise when there is a disruption to the normal sequence of events of the electrical cycle. These disruptions may be due either to abnormalities in the pulse formation or in the electrical conduction. The abnormalities show up on the ECG signal as irregularities or anomalous events.

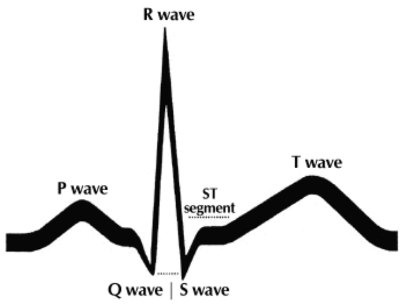

Figure 1 The ECG [10]

The ECG is usually represented on grided (1x1mm) paper. In the horizontal direction each grid represents 0.04 seconds whilst in the vertical direction each grid represents 100μV. The recording of the ECG is done by the use of the standard 12-Lead configuration, where the leads are grouped as the Limb (I,II,III), augmented Limb (aVL, aVR, aVF) and the precordial (V1,V2,V3,V4,V5,V6). The Limb and Augmented Limb leads are in the same axis on the front of the torso, whereas the precordial Leads move around the cardiac location from the front to the back of the torso.

1.2 The P-wave

The P-wave represents the depolarization of the atrium and is most prevalent in Lead II. The accurate identification and analysis of the P wave is of vital importance in early identification and diagnosis of many cardiac arrhythmias [10]. The P wave and PR interval are indicative of atrial fibrillation and other conduction problems in the atrium [10]. Abnormal P wave morphology is also indicative of atrial enlargement and congenital heart disease. The P wave can also become less pronounced with age and therefore more difficult to read and diagnose which can make identification of the P wave, and consequently, diagnosis extremely difficult. Due to its relatively low amplitude, close proximity to the QRS complex and subtle changes in morphology, the P-wave presents a challenging problem in terms of identification, extraction or enhancement. This has lead to the popularity of non-linear signal processing techniques as a possible means of solving this problem.

2 Processing Techniques

Current linear processing techniques are beginning to show limitations in their applications to the processing of time-series data, in particular biomedical signal analysis. There also exist many inherent limitations of linear processing techniques, such as the inability to discriminate between two signals of similar frequency content but difference phase information [8]. This has lead to increased interest in non-linear processing techniques such as state-space dynamics and representation, neural network analysis and non-linear filtering. Linearity is the property displayed by a system when the output of the system is directly proportional to the system input. This property is reliant upon the functions of the components of the system being proportional to the overall system, of course this is unlikely to occur in any natural system [9].

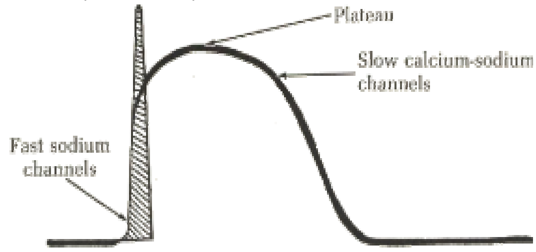

Figure 2 AV action potential (p 24 Boudreau-Conover [10])

Instead the components of the system are more likely to interact with each other and, from these interactions, produce outputs that are not proportional, and sometimes unrelated, to the original input, therefore creating a nonlinear system. For example the cardiac conduction cycle has nonlinear dependencies based upon the excitation patterns of the calcium channels present in the cardiac tissue, as shown in Figure 2. The different channels have differing conductivities and therefore produce a nonlinear reaction to their initial excitation. From this we can see that there is cause for examining the non linear dependencies of the ECG by using means other than linear time-series analysis. The obvious first choice for non-linear analysis is to examine the ECG in the frequency domain. This section examines the commonly used non-linear techniques of Wavelet processing (frequency domain), Artificial Neural Networks (classification/detection), numerical analysis including non-linear transformations, and non-linear dynamics analysis (Phase-Space portraits).

2.1 Wavelet Processing

Wavelet Processing techniques for signal processing and analysis have been well documented and widely employed for many applications including the processing and visualization of Electrocardiogram signals [11,12, 13, 14, 15]. The wavelet transform allows for the transform of linear time-series data into time-frequency levels. The continuous wavelet transform (CWT) represents the original data in a continuous frequency scale along a time axis. The cwt can be viewed as an image where the luminance or colour represents the amount of energy contained in a particular frequency at a particular time as shown in figure 3.

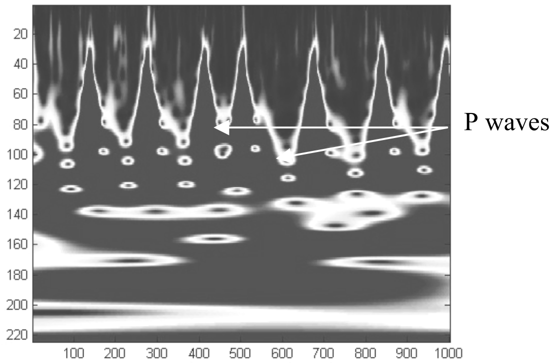

Figure 3 CWT image of an ECG signal [15]

The p-waves become evident in Figure 3 as clearly identifiable concentrations of energy preceding the QRS complex peaks as marked. Of more use in automated analysis and compression is the discrete wavelet transform. The DWT transforms the time series data into discrete time-frequency band coefficients. This allows analysis of the energy distribution of a signal of discrete narrow band frequencies.

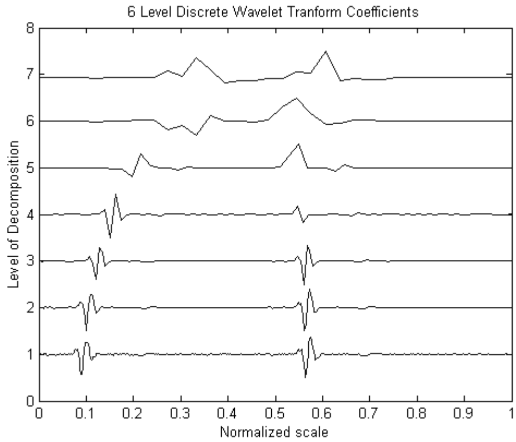

Figure 4 6-level DWT of 2 seconds of ECG data

In Figure 4, level one represents the upper half band of the original signal’s frequency spectrum; each remaining lower band is split in half again so that the coefficients represent the upper half of each band. Level 7 represents the final lower half of band 6 and the coefficients of this level are called the detail coefficients, the other bands are known as the approximation coefficients. The characteristic spikes present in the first three levels are related to the original QRS complex. By applying a threshold to these levels, thereby suppressing coefficients of low amplitude, peak detection can be applied to accurately detect and locate the QRS complex in the original signal. These QRS locations can then be used to suppress the QRS segments and search for other characteristics such as the P wave. The DWT also allows for reconstruction of the original signal from the detail and approximation coefficients which can be exploited for applications such as QRS detection [11] and data compression [14].

2.2 Artificial Neural networks

Artifical Neural Networks (ANN) are commonly used in signal processing for the classification, recognition, and detection of a required pattern. ANNs mimic the biological neural process, such as the brain, by modelling the neurological information processes by means of a mathematical paradigm [16]. In the last 50 years much work has been done applying ANN to practical applications and improving ANN structure for more effective and adaptable learning allowing for wider application of the technique [17].The common component of any Artificial Neural Network (ANN) is the McCulloch Pitts' Neuron Model [18] as shown in figure 5.

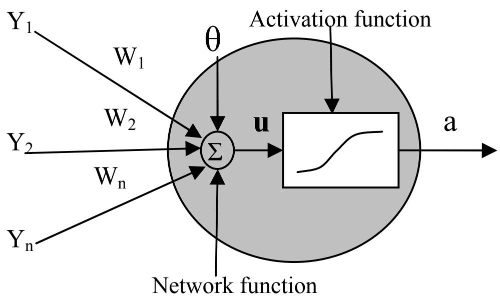

Figure 5 McCulloch and Pits neuron model [16]

The network function produces a weighted combination (**u**) of the input to the neuron, (Y_1, Y_2, Y_3….) which are usually the output of some nonlinear transformation of an original data set. Common nonlinear input transformations are covered in section 2.3. The network function may be either a linear summation or non-linear function; alternatively the network function can be the product of all the weighted inputs.

Equation 1 neuron output

$$a = f(u)$$

The actual function employed by the activation function can be graded slopes, thresholds or linear slopes; each has advantages for specific applications.

The main application for them in the ECG problem is in ECG characteristic identification or pattern recognition. The simplest method of implementing a neural network for pattern recognition makes use of the Perceptron network. The perceptron is a variation on the original neuron model where the weighting vector, $\underline{w} = \{w_1, w_2 \ldots w_N\}$ is indicative of the desired recognition pattern. In this manner, if the input data series, $\underline{x} = \{x_1, x2\ldots\ x_N\}$, is similar to the weighting vector then their inner product should exceed an appropriately chosen threshold, determined by the activation function which is a discrete threshold function.

$$u = \sum_{j=1}^{N} Y_j . W_j + \theta$$

$$a = \{ \begin{matrix} 0 & u < 0 \\ 1 & u > 0 \end{matrix}$$

Increase size of {

Where a=1 would be a detection.

A comparison of several ANN structures and learning algorithms for ECG classification in [19] found that a static ANN with principal component analysis input had the fastest training time when using a backpropagation training method. The relative simplicity of the backpropagation (BP) networks is an attractive feature and in addition to analysis of the ECG, BP networks can be adapted to also perform classification within the one network [20].

2.3 Numerical Analysis Techniques & Nonlinear Transformations

Many numerical transforms are used as the nonlinear transformation input to neural networks. These techniques are used to transform the data to provide more meaningful information sets for analysis. The following two techniques are principal component analysis and singular value decomposition. Both are common techniques used to efficiently represent data sets; however the discrete wavelet transform is also commonly used [39].

2.3.1 Principal Component Analysis (PCA)

PCA is a mathematical procedure whereby a dataset containing interrelated variables is collated into a correlation or covariance matrix. It is from this matrix the principal components are calculated such that the first component describes as much of the variability of the original data as possible, with each successive component describing as much of the residual variability as possible. In this way the dimensions of the original data set can be reduced by removing and expressing the interrelated data components in a more succinct form [21].

The application of PCA in signal processing can roughly be categorized into two applications where the goal is to reduce the original dimensions of data for compression or to extract meaningful characteristics of the original data so as to perform some form of classification or recognition. In terms of data dimensionality reduction PCA provides the optimum linear reduction and is often referred to as the Karhunen-Loeve expansion [16]. The PCA technique is primarily used as a pre-processor for Neural Networks which perform detection and classification, such as human identification through ECG analysis [22], face recognition [23] and Ischemia detection [24]. The latter uses a nonlinear PCA technique which will be discussed in section 2.3.3.

2.3.2 Singular Value Decomposition

Singular Value Decomposition (SVD), or eigenvalue decomposition, is a mathematical transform that can be used to reduce the amount of data required to represent a matrix. This allows for the efficient coding of images by means of lossy compression. The SVD of a matrix A of dimension (n x p), where n is the number of rows and p the number of columns, is given in equation 5:

Equation 2 Singular Value Decomposition

$$A = USV^T$$

where;
U and V are (n x r) and (p x r) matrices respectfully.
S is a (r x r) diagonal matrix.

S is an (r x r) diagonal matrix whose leading diagonal elements are known as the singular values. The matrices *U* and *V* are the left and right single value matrices and whose columns are orthonormal such that $U^TU = I_r = V^TV$. As previously mentioned SVD is used in signal processing applications primarily such as image compression [8] and noise removal by means of nonlinear filtering [25],[26] and also as an efficient algorithm for calculating principal components [21].

2.3.3 Nonlinear PCA

As mentioned earlier nonlinear PCA can provide a nonlinear mapping of the original data source by means of replacing the original data set x with a function of x [21]. There exist two models for creating a nonlinear PCA system, symmetric and hierarchical, of which the latter provides better classification through higher discrimination of significant features [27] [28] [29]. In [24] the non linear principal component analysis (NLPCA) is used to group and classify the nonlinear clusterings of normal and abnormal ischemic beats in a feature space. NLPCA provides a much more accurate characterisation of the clusterings then a normal PCA does.

2.4 Nonlinear Dynamics analysis through Phase-Space portraits

Nonlinear Dynamics is the term applied to the study of the complex behaviour of relatively simple systems [34] and as has been previously described in section 2 the heart represents such a system. The term state or phase space refers to the analysis of a system by its change in states and current state. This is often done by means of phase portrait representation as shown in figure 7. When applied to visualization of the ECG the phase portrait offers simple intuitive representations of the signal and easily highlights time varying properties of the ECG, such as increase in segment or complex duration. The true Phase-Space of an ECG cannot be reconstructed solely from the ECG signal; however a pseudo Phase Space, which is equivalent to the original phase space, may be reconstructed by the use of one of several methods. It can simply be presented as the original time series signal related to the first derivative of the time series, although in [30] the phase space derivation also includes a lag or delay term. The delay, or lag, used can be calculated using several methods; method of delays, trial and error, mutual information [31,32]. The most computationally simple method of calculating the optimum delay is the Takens' method of delays which ensures that the phase portrait is adequately decipherable [31]. That is, the ECG attractors are well spread allowing each characteristic loop of the ECG to be easily identified by visual inspection, such that the QRS loop is broad enough not to interfere with the other characteristic loops of the ECG. The attractors of a system describe its long term behaviour from starting conditions and maybe point, limit cycle (periodic) or chaotic (strange) in nature. Point attractors describe the system in equilibrium from any set of starting conditions after all motion has ceased. Limit cycle attractors describe the steady state oscillations of a system after the transients have subsided, however this state of steady oscillation may alter dependent upon initial conditions of the system. Finally chaotic attractors describe the aperiodic behaviour of the system [33] and their trajectories exhibit exponential divergence away from nearby paths [34].The mapping of the ECG time series data into this feature space phase portrait in Figure 7 shows each waveform of the ECG clearly distinguished and immediately apparent. Identifying the underlying attractors of the system means that the ECG can be still be a accurately analysed by such means as NLPCA [24] in the presence of temporal noise [34].

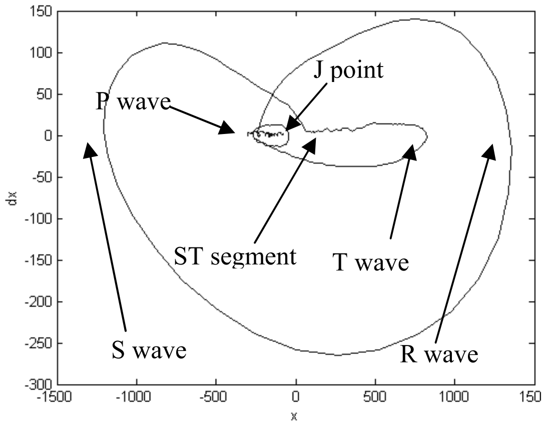

Figure 6 Phase Portrait of normal sinus rhythm

3 P Wave Detection Techniques

The common difficulties identified in relation to the identification of the P-wave are susceptibility to noise, low amplitude and merging with T waves in periods of tachycardia. The common solution to this problem is to implement some form of QRS complex suppression and then "search" for the P-wave. The methods used range from standard linear processing techniques to more complex applications of several non-linear processing techniques all with varying levels of success.

3.1 Linear Adaptive Signal Processing

In the field of Digital Signal Processing the term "adaptive" generally applies to systems which take into consideration that the processes which are being analysed are non-stationary; that is the statistical characteristics of the processes vary with time and therefore the system must vary accordingly. In general non-adaptive systems work quite well when used for processing Electrocardiograms due to the generally consistent frequency content of the signal or pseudo-stationary behaviour of the ECG as a process; see Figure 8.

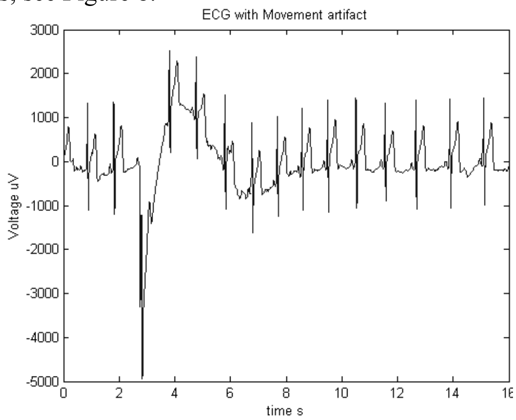

Figure 7 ECG signal containing movement artefact

The Power Spectral Density (PSD) estimate is a representation of the distribution of a signal across the original signal's frequency spectrum. The PSD shown in Figure 8 was calculated by cutting the ECG signal into blocks then performing the Burg PSD approximation on each block. The inadequacies of linear filtering become apparent when this approximation does not hold; the movement artefact present in Figure 7 is clearly evident in the PSD graph as the plot which deviates from the envelope created by all other frames of the signal. The noise is now spread across the frequency range inhabited by the normal ECG.

A simple first order filter accompanied with decision rules is implemented in [35] to detect onset of P and end of T waves. This algorithm has the advantage of being user defined, that is a cardiologist can initiate the algorithm with their definition of P wave onset and the end of the T wave. From these initial conditions an adaptive filtering constant μ is approximated and the ECG is filtered with a first order adaptive system utilizing this constant.

$$H(z) = \frac{1 - z^{-1}}{1 - (1 - \mu).z^{-1}}$$

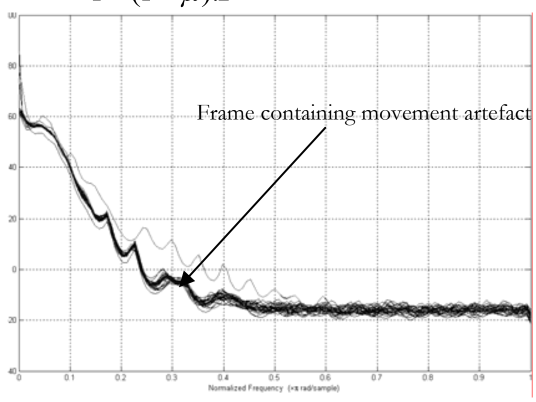

Figure 8 PSD estimate of ECG containing movement artefact

A second iteration of the constant approximation and filtering is performed and the first order discrete time derivative is taken. The points corresponding to zero are judged to be the P-wave onset and T-wave offset. A real-time filter based approach to ECG characteristic detection is developed and implemented in [36]. This method performs real-time analysis of the ECG and detects the onset and offset of each characteristic beat. The QRS complex is logically detected first and upon successful detection a search window is defined for the T wave which must follow the QRS. In the resulting window left between the detected T wave and QRS complex the P wave is searched for using the following rules.

The P wave is defined as:

- A positive slope followed by a negative slope
- Where both slopes have a magnitude greater than 0.004mV/s

The remaining characteristics such as RR, ST, PQ and QT intervals are derived from the wave locations as are the onset, peak and offset of each P wave detected.

3.2 Wavelet Techniques

Wavelet techniques for processing physiological signals, and in particular the ECG, have been well documented. Wavelet transforms are used in robust QRS detection to highlight the QRS complex and allow for easy location of each beat, when applied to the P-wave detection problem they are generally used to locate and suppress the QRS complexes [37]. The wavelet coefficients can also be used as the input to more complex pattern recognition techniques such as Hidden Markov Models [38] and back propagation neural networks [39]. In [38] a method of segmenting each beat in the ECG in order to extract and classify the P-waves is presented and discussed. The method is used to identify patients who are susceptible to Atrial Fibrillation (AF).

The steps involved in this algorithm are as follows

- Band limit ECG 0.01 – 40 Hz
- 4 level, multi resolution analysis using Haar wavelet transform.
- QRS detection and suppression
- Beat Segmentation by applying a Hidden Markov Model to each resolution level and correlating the results.

In [38] the ECG is first segmented into its various components so that the hidden Markov Model's states for the ECG can be determined. This creates a logical yet flexible pattern recognition model. The wavelet pre-processor is proven to be effective in decreasing the time or number of iterations required to train the ANN shown in [39].

3.3 Artificial Neural Networks

As previously mentioned ANN are primarily used in signal processing applications for pattern classification, pattern recognition or detection. They can then be thought of a as a dedicated form of classification where there exists only two output states; detection = true or false. However this binary definition is not held by Neuro-fuzzy networks [40] which employ a probabilistic approximation in terms of classification. The reduced emphasis on producing a definitive answer means that processing time needed to produce a result can be reduced; however this in turn means a reduction in accuracy of detection or rejection. To summarise, the Neuro-fuzzy network provides a quick and rough result based upon statistical properties obtained through training. The morphology of the P wave is also a factor which can alter detection rates and therefore the classification of this morphology would be of benefit to an ANN. Classification of the P wave into four groups, positive, negative, bi-phasic and M-shaped is achieved in [41] where the salient feature is the implementation of an asymmetric basis function which the authors claim produces higher classification results by improving the rate of classification for biphasic and M shaped P waves.

4 Comparison of P wave Detection methods

Most algorithms were tested using the MIT-BIH arrhythmia database; however a specific database dedicated to atrial fibrillation was compiled and used in [38], a database of healthy test patients was used in [40]. Most methods listed more than one measure of evaluation, such as sensitivity, positive predictivity, false detection, etc.

The sensitivity quality was chosen as the common denominator by which to compare the collated methods and the relative values for each method appear under the detection accuracy heading in Table 1. Unless otherwise specified these values relate to the sensitivity of the algorithm stated in the respective papers. Sensitivity is the proportion of actual positives that were predicted that way; here it describes the percentage of true positive P waves detected.

In some instances this information was not available, as in [35] where the positions of the detected P waves were compared to the onset and offsets as determined by a cardiologist. The highest rates of detection were associated with the adaptive filter system [36] and a neural network analysis system using a discrete wavelet transforms pre-processor [42]. In the case of the Adaptive filter proposed in [35] a cardiologist is required to mark the onset of the P waves and offset of the T waves, whilst this is useful due to the highly ambiguous nature of the P wave onset and the individual interpretation of it, it means that the system can not perform automated detection of the P waves. Automated detection would be desirable for long term monitoring but not essential for clinical monitoring. The authors also claim that baseline drift has no effect on the performance of the adaptive filter system. The flexibility of the Neural Networks to be applied to not only detection but classification of the P wave is an advantage over the adaptive signal processing techniques of [36] [35] which are dedicated solely to the detection of the P-wave. Results show that the choice of pre-processor has a significant impact upon the effectiveness and speed of the detector.

Table 1 Comparison of P wave Detection methods

System	Data base	Comple xity	Positi on Accuracy	Detectio n Accuracy
Adaptive Signal Processing [35]	Healthy Database &. MIT-BIH segments containing 50 Beats	Simple, requires user input definitions, Training	2.31 +/- 4.57 ms [1]	-
Real time ECG characteristic detection with moving average filters [36]	QT database (MIT-BIH was used solely for QRS)	Simple moving average filters and rule book	-	69.3 % (onset) 73.8 % (peak) 77.6 % (offset) 73.6 % (mean)
Neural Network with Asymmetric Basis Function [41]	MIT-BIH recordings	Requires Training	-	Performs classification of the P-wave
Neuro fuzzy network [40]	Independe nt Data, Healthy test patients.	Requires Training	-	83.25%,
Wavelet transform and HMM analysis [38]	Brest University AF database		-	65-70%,
Wavelet decomposition for ANN analysis [42]		Requires Training	-	92% (relative to 96.19% QRS complexes detected).

The wavelet pre-processor applied to both the Neuro fuzzy network in [40] and the backpropagation ANN in [41] reportedly improves the speed of the networks training with both networks returning high levels of correct identification. The Wavelet transform fed HMM [38] in comparison has a much lower sensitivity for an increased level of complexity. Nonlinear dynamical analysis of the ECG has briefly been explored in [30] and [31] however a practical means of applying the dynamic analysis. The high degree of separation of the individual ECG waveforms provided by Figure 6 would suggest that the application of NLPCA to the phase space, similar to that which is applied to feature space data in [24], would provide a novel means of ECG characteristic identification, in particular the identification and enhancement of the P wave.

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