Level-set Methods in Computer Vision and Medical Imaging

Baba C. Vemuri
Department of Computer & Information Sciences & Engineering,
University of Florida, Gainesville, FL 32611
(email:vemuri@cise.ufl.edu)

Level-set methods gained popularity in Computer Vision, Image Processing and Medical Imaging after their inception in these fields by Malladi et.al., [11, 12, 13], who primarily applied it for the image segmentation problem. They have however since been used for other tasks such as selective image smoothing, image registration, tracking and others. Each of these problems will be briefly discussed in the following along with their existing solutions in a level-set framework.

The problem of image smoothing is ubiquitous in the fields of Computer Vision, Image Processing and Medical Imaging. One of the most popular image smoothing techniques used widely to date is the linear filtering with a Gaussian. It is well known that filtering an image with a Gaussian is equivalent to solving the linear diffusion equation (a parabolic PDE) with the image being filtered as the initial condition. The problem with linear filtering is that most of the desired details are obliterated. Over the past decade, PDE-based image processing has become quite popular due to the rigorous framework it provides for analyzing the problem and its solution as well as the amenability to the use of efficient numerical techniques in achieving a solution. In the Vision community, Perona and Malik introduced the first edge-preserving nonlinear diffusion filter by making the coefficient of diffusion in the linear filter depend on the image being smoothed. This was done by using the equation $I_t = div(c(\nabla I)\nabla I)$, where I is the image to be smoothed and I_t describes its evolution over time, and $c(\nabla I)$ is a decreasing function of ∇I. Nitzberg and Mumford [14] and Alvarez *et al.* [9] recognized the ill-posedness of the Perona-Malik diffusion and proposed modifications to overcome the same. Since then, several nonlinear diffusion methods have been developed and a good account of these can be found in [6, 9, 17, 23] and references therein. All of these are nonlinear models and differ in the diffusivity coefficient and/or the diffusion term. Some of them are also supplemented with a reactive term. Another popular framework for image smoothing is the total variation or TV-norm framework pioneered by Rudin et.al., [16] and further developed by Strong and Chan [19]. The total variation methods yield nonlinear diffusion equations that are always derived from variational principles using the TV-norm. In [17], Shah developed a common framework for image de-noising and segmentation. In this work, a new segmentation functional was developed which lead to a coupled system of PDEs, one of them performed nonlinear smoothing of the input image and the other smoothed an "edge strength" function. Shah [17] demonstrated that all the existing curve evolution and anisotropic diffusion schemes reported in literature can be viewed as special cases of his method. In [5], Chen et. al., a nonlinear diffusion equation supplemented with reactive terms for achieving edge preserving smoothing was presented. *Some of these methods involved level-set curvature-based smoothing terms. All of these methods are primarily applicable to the selective smoothing of scalar valued images.* Smoothing vector valued images has been less popular than the scalar valued image data sets. Vemuri et.al., [21] introduced a novel and efficient weighted total variation (TV) norm based image smoothing scheme and applied it for smoothing very high dimensional vector-valued image smoothing used in the construction of diffusion tensor images of the human brain and the spinal cord. For other vector-valued image selective smoothing techniques, readers are referred to a survey by Weickert [23] and also [2].

In the context of image segmentation or shape segmentation from image data, numerous researchers in the image analysis community over the past several years have focussed on this problem. Since the inception of active contours and surfaces a.k.a. snakes, in the vision & graphics communities by Kass *et al.* [7], these elastically deformable contours/surfaces and variants thereof have been widely used for a variety of applications including boundary detection and representation, motion tracking etc. of shapes of interest. A viable alternative to the snakes model was proposed by Malladi *et al.* [11, 12, 13] and Caselles *et al.* [3]. These models are more general and intrinsic than the traditional snakes [7] and are based on the theory of curve evolution and geometric flows. Automatic changes in topology can be handled in a natural way in this modeling technique, by implementing the curve evolution using level-set embedding schemes. A generalization of this model was later proposed simultaneously by Caselles et. al., [4] and Kichenassamy et. al., [8]. The generalization also known as the *geometric active contours* depicted the link between the Kass et. al., [7] snakes and the *geometric active contours* a.k.a. *geodesic or geometric snakes*. For details on the theory of curve/surface evolution and its level-set implementation, the reader is referred to [3, 11, 13, 4, 8, 20] and references therein. Geometric active contours and their recent variants ([18, 15, 10, 24]) – some of which use region gray level statistics – are quite successful in recovering shapes from medical as well as non medical images. They are relatively insensitive to the initialization problem, do not have too many user specified parameters and can handle arbitrary topologies in an elegant manner. Facilitating the incorporation of shape priors in the geometric active contour/surface models has been achieved using recently introduced concepts of deformable pedal curves/sources in Vemuri et.al., [21].

Finally, in the context of image registration and tracking applications, level-set based approaches have been proposed by Vemuri et. al., [22], Bertalmio et.al., [1], Paragios et.al., [15]. If we consider the image registration problem from the point of curve/surface evolution, registering two given intensity images can be intuitively thought of as determining ways to evolve the level-sets of the intensity function of one image (say the source image) into level-sets of the intensity function of another image (call it the target image). So, given two images $I_1(X)$ and $I_2(X)$, we want $I_1(X)$ to evolve into $I_2(X)$. The evolution can thus be written down as $I_t(X,t) = S\|\nabla I(X,t)\|$ *with* $I(X,0) = I_1(X)$. Since this evolution should only stop when image $I(X)$ changes from $I_1(X)$ to $I_2(X)$, we need to include this stopping mechanism in the speed term. Therefore the natural choice for it will be $S = I_2(X) - I(X,t)$. In all the registration applications however, we need to determine this geometric transformation explicitly between the two images and this can be achieved via the use of the following equation which is derived in a similar fashion: $\vec{V}_t = [I_2(X) - I_1(\vec{V}(X))]\frac{\nabla I_1(\vec{V}(X))}{\|\nabla I_1(\vec{V}(X))\|}$ with $\vec{V}(X,0) = \vec{0}$. Where $\vec{V} = (u,v)^T$ is the displacement vector at X and the operation $\vec{V}(X) = (x-u, y-v)^T$. Note that, this equation assumes that the intensity/brightness is the same at corresponding points between the two images being registered. One of the salient features of this registration scheme over existing methods in literature is the computational efficiency and its simplicity. It takes approximately 3mins. to register two large volumetric images of size (256,256,120) each on a single R10000 processor ONYX.

In this talk I will give an concise overview of the problems as well as some of the solution methods briefly discussed above.

References

[1] M. Bertalmio, G. Sapiro, and G. Randall. Morphing active contours. In *Proc. Scale-Space Conference*, Greece, September 1999.

[2] Eds: V. Caselles, J. M. Morel, G. Sapiro, and A. Tannenbaum. Special issue on pdes and geometry-driven diffusion in image processing and analysis. *IEEE Trans. on Image Processing*, 7(3), 1998.

[3] V. Caselles, F. Catte, T. Coll, and F. Dibos. A geometric model for active contours in image processing. *Numerische Mathematik*, 66:1–31, 1993.

[4] V. Caselles, R. Kimmel, and G. Sapiro. Geodesic active contours. In *Fifth International Conference on Computer Vision*, pages 694–699, 1995.

[5] Y. Chen, B. C. Vemuri, and L. Wang. Image denoising and segmentation via nonlinear diffusion. *Computers and Mathematics with Applications*, 39(5):131–149, 2000.

[6] F.Catte, P.L. Lions, J.M. Morel, and T.Coll. Image selective smoothing and edge detection by nonlinear diffusio. *SIAM Journal of Numerical Analysis*, 29:182–193, 1992.

[7] M. Kass, A. Witkin, and D. Terzopoulos. Snakes : A active contour models. *Int. Journal of Computer Vision*, 1:321–331, 1987.

[8] S. Kichenassamy, A. Kumar, P. Olver, A. Tannenbaum, and A. Yezzi. Gradient flows and geometric active contour models. In *Fifth International Conference on Computer Vision*, pages 810–815, 1995.

[9] L.Alvarez, P. L. Lions, and J. M. Morel. Image selective smoothing and edge detection by

nonlinear diffusion ii. *SIAM J. Numer. Anal.*, 29:845–866, 1992.

[10] L. M. Lorigo, O. Faugeras, W. E. L. Grimson, R. Keriven, R. Kikinis, and C. F. Westin. Codimension 2 geodesic active contours for mra segmentation. In *Proc. of the XVI Intl. Conf. on Info. Processing in Medical Imaging*, pages 126–139, June/July 1999.

[11] R. Malladi, J. A. Sethian, and B. C. Vemuri. A topology independent shape modeling scheme. In *SPIE Proc. on Geometric Methods in Computer Vision II*, volume 2031, pages 246–256,. SPIE, July 1993,.

[12] R. Malladi, J. A. Sethian, and B. C. Vemuri. Eolutionary fronts for topology independent shape modeling and recovery. In *Third European Conference on Computer Vision*, volume 1, pages 1–13. Springer-Verlag, May 1994,.

[13] R. Malladi, J. A. Sethian, and B. C. Vemuri. Shape modeling with front propagation : A level set approach. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 17(2):158–175, 1995.

[14] M.Nitzberg and T.Shiota. Nonlinear image filtering with edge and corner enhancement. *IEEE Trans. on PAMI*, 14:826–832, 1992.

[15] N. K. Paragios and R. Deriche. A pde-based level-set approach for detection of moving objects. In *Proc. of the IEEE Intl. Conf. on Compu. Vision*, pages 1139–1145, Jan. 1998.

[16] L. I. Rudin, S. Osher, and E. Fatemi. Nonlinear image filtering with edge and corner enhancement. *Physica D*, 60:259–268, 1992.

[17] J. Shah. A common framework for curve evolution, segmentation and anisotropic diffusion. In *IEEE Conf. on Computer Vision and Pattern Recognition*, San Francisco, CA, 1996.

[18] K. Siddiqi, Y. B. Lauziere, A. Tanenbaum, and S. W. Zucker. Area and length minimizing flows for shape segmentation. *IEEE Trans. on Image Processing*, 7(3):433–443, 1998.

[19] D. M. Strong and T. F. Chan. Relation of regularization parameter and scale in total variation based denoising. In *Proc. IEEE Workshop on MMBIA*, June, 1996.

[20] H. Tek and B. B. Kimia. Image segmentation by reaction-diffusion bubbles. In *Fifth International Conference on Computer Vision*, Boston, MA, 1995.

[21] B. C. Vemuri, Y. Guo, and Z. Wang. Deformable pedal curves and surfaces: Hybrid geometric active models for shape recovery. *Intl. Journ. of Computer Vision*, 44(2):137–155, 2001.

[22] B. C. Vemuri, J. Ye, Y. Chen, and C. M. Leonard. A level-set based approach to image registration. In *Proc. of the IEEE MMBIA*, 2000.

[23] J. Weickert. A review of nonlinear diffusion filtering. In B. ter Haar Romney *et al.*, editor, *Scale-space theory in computer vision*, volume 1252, pages 3–28, Springer-Verlag, 1997.

[24] A. Yezzi, A. Tsai, and A. Wilsky. A statistical approach to snakes for bimodal and trimodal imagery. In *Proc. of the IEEE Intl. Conf. on Compu. Vision*, pages 898–903, Sept. 1999.