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Correction of a 3D Object Reconstructed from a Single Image

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Abstract

Previous methods for 3D object reconstruction from single line drawings assume that a drawing depicts a 3D object in a parallel projection. These methods provide a convenient way for a user to reconstruct the 3D structure of an object. However, when these methods are applied to generate 3D photo-realistic objects from images, the objects reconstructed are distorted because such objects are formed in images under projective projections. In this paper, a method based on perspective geometry is proposed to remove the distortion. The method mainly consists of two tasks: compensating the values of z-coordinates of the vertices of a distorted 3D model and then recovering the normalized model. The experimental results show that the method can correct the distortion.

1. Introduction

3D object reconstruction from single line drawings has been proposed in several papers [1-4]. An application of this reconstruction to generate 3D photo-realistic objects is presented in [4]. Given a 2D image, we can draw lines with a mouse over interested parts in the image, and then reconstruct the 3D surfaces from the line drawing. After mapping the needed texture from the image onto the surfaces, we obtain a photo-realistic 3D object. However, the 3D object reconstruction from line drawing (*3DORLD*) algorithm in [4] assumes that the line drawing depicting an object is in a parallel projection. This assumption gives rise to distortion in the reconstructed 3D object, because objects in an image are captured in a perspective projection. For example a cube in an image, after being reconstructed, will become a non-rectangular prism.

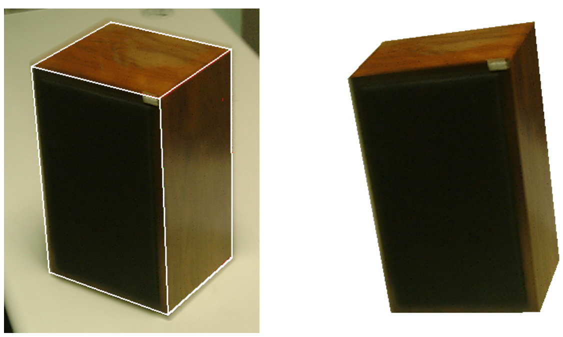
In this paper, we propose a method to correct such distortion. In this method, the line drawing obtained from an image is regarded as the parallel projection of another object distorted from the original one. We, first reconstruct the 3D distorted model from the line drawing using the 3DORLD method in [4]. Then the z-coordinates of the vertices of the distorted model are compensated. Finally, we nomalizedly recover the coordinates of the original object from the compensated distorted model. We also give a criterion of line parallelism error to measure the performance of the proposed method. The experimental results show that the proposed method is effective.

This paper is organized as follows. Section 2 describes the problem of the 3D reconstruction distortion when generating 3D photo-realistic objects from images by the 3DORLD method. The proposed correction method and the line parallelism error are given in Section 3. The experimental results are presented in Section 4. Finally Section 5 gives the conclusions.

2. Distortion of 3D objects reconstructed by 3DORLD

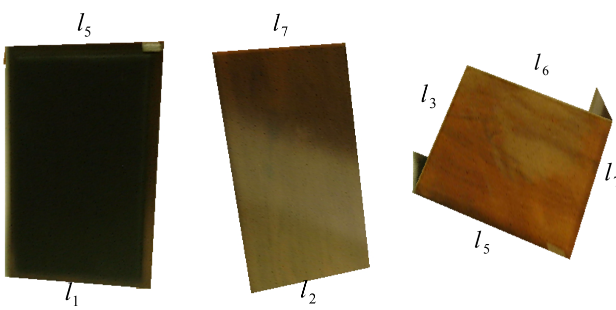
The 3DORLD method for the reconstruction of the 3D surfaces from a given line drawing consists of two steps: (a) identifying the face configuration from the line drawing and (b) reconstructing the 3D geometry based on the faces found. In [4], the face identification is formulated as a maximum weight clique problem, based on (a) the observation that a human tends to choose a face configuration in which there are as many edges as possible, and (b) the face adjacency theorem which states that two adjacent planar faces may coexist in the same object if and only if their common edges are collinear [3]. Solving the maximum weight clique problem, we can find the maximum weight clique that corresponds to the face configuration of a drawing. For the 3D reconstruction problem, the 3DORLD method [4], uses a set of image regularities [1-3] to establish the relations between a 2D line drawing and a 3D object whose shape should accord with the human interpretation of the drawing. The image regularities include face planarity, line parallelism, isometry, corner orthogonality, skewed face orthogonality, and minimum standard deviation of angles. Each of them corresponds to a constraint and all the constraints are combined into an objective function. The 3D coordinates of the vertices of a line drawing can be derived from minimizing this function, and thus the surfaces of the object represented by the line drawing are obtained.

As mentioned in the last section, the 3DORLD method assumes that a line drawing describes an object in a parallel projection. However, an object in an image is captured in a perspective projection. When the 3DORLD method is applied to generate photo-realistic 3D models, shape distortion is caused. Fig. 1(a) shows a line drawing imposed on a speaker in an image. The original shape of the speaker is a rectangular block. With the 3DORLD method, we can generate the 3D object as shown in Fig. 1(b), which is displayed from another viewpoint. We can notice the reconstruction distortion easily when the object is further rotated. Fig. 2 shows some examples. We can notice that the rectangular sides of the speaker are converted into kites. Any pair of lines in $\{(l_1, l_5), (l_2, l_7), (l_3, l_7), (l_6, l_5)\}$ are not parallel in 3D space, while they are supposed to be. These distortions result from the assumption in the 3DORLD method that the 2D line drawing is a parallel projection of a 3D object. Note that all the 3D reconstructed model are shown in a parallel projection in this paper.



(a) (b)

fig. 1. (a) A line drawing plotted over a speaker. (b) 3D model reconstructed by the 3DORLD method.



(a) (b) (c)

Fig. 2. Three rectangular sides of the box are converted into kites.

These examples reveal the shortcoming when the 3DORLD method is used to generate 3D photo-realistic objects from images. For the same focus length, the shorter is the distance between the camera center and the original 3D object, the more remarkable the distortion is.

3. Correction of the distortion

3.1. Obtaining the 2D perspective projection of a 3D object

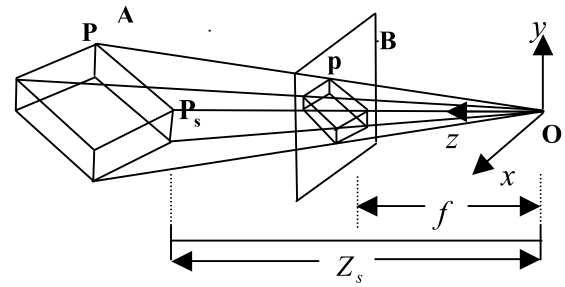


Fig. 3. Relationship between the 3D object **A** and its perspective projection **B**.

Suppose that an image is taken by a pinhole camera, the origin **O** of a 3D world coordinate system coincides with the camera center, and the x and y axes of the world coordinate system are parallel to the x and y axes of the image coordinate system respectively (see Fig. 3). For simplicity, we also assume that the unit of the world coordinates system is the same as that of the image coordinate system and there is no lens distortion. Then the perspective projection from the world to image coordinates can be described as [5]:

$$Z\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{1}$$

where f is the focus length of the camera, $P(X,Y,Z)$ is a vertex of the original object **A** and $\mathbf{p}(u,v)$ is the corresponding vertex of **P** in the 2D object **B** which is the perspective projection of **A** in the image plane, as shown in Fig. 3. Let $\mathbf{P_s}$ be the vertex whose z-coordinate Z_s is the smallest among all the z-coordinates of the vertices of **A.** From (1), the coordinate (u,v) of **P** can be computed by

$$\begin{aligned} u &= X \cdot (f / Z) \\ v &= Y \cdot (f / Z)\,. \end{aligned} \tag{2}$$

3.2. Obtaining the 2D object B in another way

B, which is the perspective projection of **A,** can be replaced by the parallel projection of **A′** which is another 3D object obtained by distorting the object **A**. Let $\mathbf{P}'(X',Y',Z')$ be a vertex in **A′** and $\mathbf{P_s}'(X_s', Y_s', Z_s')$ be the vertex in **A′** corresponding to $\mathbf{P_s}$ in **A**. First, we distort **A** by a transform $\mathbf{T_1}$ to obtain the distorted 3D object **A′** (see Fig. 4.).

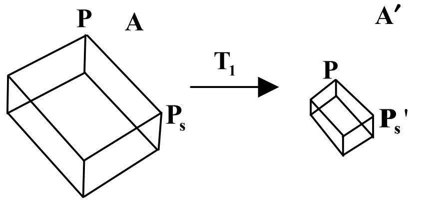


Fig. 4. Distorting the original object A to obtain A′.

$\mathbf{P'}(X',Y',Z') = \mathbf{T}_1(\mathbf{P}(X,Y,Z))$

or

$$\begin{aligned} X' &= X\cdot(f/Z) \\ Y' &= Y\cdot(f/Z) \\ Z' &= Z\cdot(f/Z_s) \ . \end{aligned} \tag{3}$$

Thus we have

$$\begin{aligned} Z_s' &= Z_s\cdot(f/Z_s) = f \\ Z/Z_s &= Z'/Z_s' = Z'/f \ . \end{aligned} \tag{4}$$

Now, we can obtain the 2D object **B′** by parallelly projecting A′ onto the image plane in the direction of $-z$ in the world coordinate system, as shown in Fig. 5. Let $\mathbf{p'}(u',v')$ be a vertex in **B′** corresponding to vertex **P′** in the distorted object **A′.** u' and v' can be computed by the affine transform:

$$\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix}$$

$$\begin{aligned} u' &= X\cdot(f/Z) \\ v' &= Y\cdot(f/Z) \end{aligned} \tag{5}$$

Comparing (5) and (2), we can see that **p** is equal to **p′** and **B′** is the same as **B**. Thus the perspective projection **B** of the original object **A** can be regarded as the parallel projection of **A′** which is another 3D object obtained by distorting the object **A**.

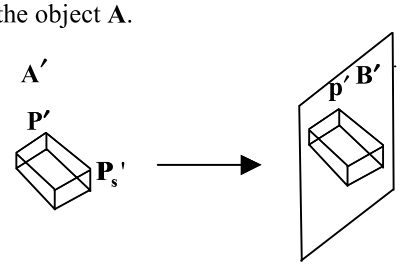


Fig. 5. Parallel projection of **A′**.

3.3. Compensation of the z coordinates reconstructed by the 3DORLD method

With a 3DORLD method such as those given by [4], a 3D object $\mathbf{A}^*$ can be obtained from **B,** which is the parallel projection of **A′** (Fig. 6). Let $\mathbf{P}^*\ (X^*,Y^*,Z^*)$ be a vertex of the distorted 3D model $\mathbf{A}^*$ and $\mathbf{P}_s^*$ be the vertex whose z-coordinate Z_s^* is the smallest among all the *z*-coordinates of the vertices of $\mathbf{A}^*$ ($\mathbf{P}_s^*$ corresponds to $\mathbf{P}_s$ in **A**). The 3DORLD method based on parallel projection ensures:

$$\begin{aligned} X^* &= X', \\ Y^* &= Y' \end{aligned} \tag{6}$$

and obtains the relative value ΔZ^* of Z^* but not the absolute coordinate Z^* where

$$\Delta Z^* = Z^* - Z_s^*, \tag{7}$$

with Z_s^* being an arbitrary value. We may assume $Z_s^* = Z_s'$ for simplicity. In general, ΔZ^* is not equal to $\Delta Z' = Z' - Z_s'$, and thus we need to find the relationship between them in order to recover the original object **A**.

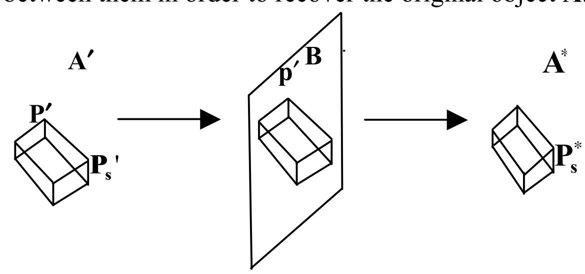


Fig. 6. Reconstruction of $\mathbf{A}^*$ from **B** with the 3DORLD method.

The 3DORLD method is a linear method in the sense that if $\tilde{\mathbf{C}}$ and $\hat{\mathbf{C}}$ are the same 2D drawings of different scales, the 3D object reconstructed from $\tilde{\mathbf{C}}$ will be the same as that reconstructed from $\hat{\mathbf{C}}$ but also of different scales. Let $\hat{\mathbf{B}}$ be the parallel projection of **A**. Then we have $\Delta\hat{Z} \cong \Delta Z$, if $\hat{\mathbf{B}}$ is obtained in a generic view [3], where $\Delta Z = Z - Z_s$ and $\Delta\hat{Z}$ ($= \hat{Z} - \hat{Z}_s$) are the relative z-coordinates of the vertices of $\hat{\mathbf{A}}$ reconstructed from $\hat{\mathbf{B}}$. Let $\tilde{\mathbf{B}}$ be a scale γ of $\hat{\mathbf{B}}$, and $\tilde{\mathbf{A}}$ be the 3D object reconstructed form $\tilde{\mathbf{B}}$. If $\hat{\mathbf{p}}(\hat{u},\hat{v})$ is a 2D vertex in $\hat{\mathbf{B}}$, then its corresponding vertex in $\tilde{\mathbf{B}}$ is $\tilde{\mathbf{p}}(\gamma\hat{u},\gamma\hat{v})$. The linear method leads to

$$\Delta\tilde{Z} = \Delta\hat{Z}\cdot\gamma \cong \Delta Z\cdot\gamma,$$

where $\Delta\tilde{Z} = \tilde{Z} - \tilde{Z}_s$ are the relative *z*-coordinates of the vertices of $\tilde{\mathbf{A}}$. When $\gamma = f/Z_s$, we have

$$\begin{aligned} \Delta\tilde{Z} &= \Delta\hat{Z}\cdot\gamma \cong \Delta Z\cdot(f/Z_s) \\ &= (Z - Z_s)\cdot(f/Z_s) = Z' - Z_s' = \Delta Z' \end{aligned} \tag{8}$$

and

$$\begin{aligned} \gamma\hat{u} &= (f/Z_s)\cdot X, \\ \gamma\hat{v} &= (f/Z_s)\cdot Y\ . \end{aligned} \tag{9}$$

Recall that, ΔZ^* is obtained from **p** (u, v) in **B,** where

$$u = (f / Z) \cdot X \ ,$$
$$v = (f / Z) \cdot Y \tag{10}$$

With (8), (9) and (10), the relation between ΔZ^* and $\Delta Z'$ is given by:

$$\Delta Z' \cong \Delta \widetilde{Z} = \Delta Z^* \cdot (Z / Z_s) \,. \tag{11}$$

Thus we can obtain the relative z-coordinate $\Delta \dot{Z}^*$ which is an approximation to $\Delta Z'$.

$$\begin{aligned}\Delta \dot{Z}^* &= \Delta Z^* \cdot (Z / Z_s) = \Delta Z^* \cdot (\Delta Z' + Z_s') / Z_s' \\ &= \Delta Z^* \cdot (\Delta Z^* \frac{(\Delta Z' + Z_s')}{Z_s'} + Z_s') / Z_s' \end{aligned} \tag{12}$$

In general, $(\Delta Z^* \cdot \Delta Z')/(Z_s^2) << 1$, hence (12) can be simplified as:

$$\begin{aligned}\Delta \dot{Z}^* &\cong \Delta Z^* \cdot (\Delta Z^* + Z_s') / Z_s' \\ &\cong \Delta Z^* \cdot (\Delta Z^* + f) / f \,. \end{aligned} \tag{13}$$

Similarly we assume $\dot{Z}_s^* = Z_s^* = Z_s'$. Thus

$$\dot{Z}^* = \Delta \dot{Z}^* + \dot{Z}_s^* \cong \Delta Z' + Z_s' = Z' \,. \tag{14}$$

We denote $\dot{\mathbf{A}}^*$ the object obtained by compensating all the *z*-coordinates of the vertices of $\mathbf{A}^*$. Let $\dot{\mathbf{P}}^*$ $(\dot{X}^*, \dot{Y}^*, \dot{Z}^*)$ be a vertices of $\dot{\mathbf{A}}^*$ where $\dot{X}^* = X^* = X'$ and $\dot{Y}^* = Y^* = Y'$ (see (6)). $\dot{\mathbf{A}}^*$ is approximately equal to $\mathbf{A}'$ due to (14) as shown in Fig. 7.

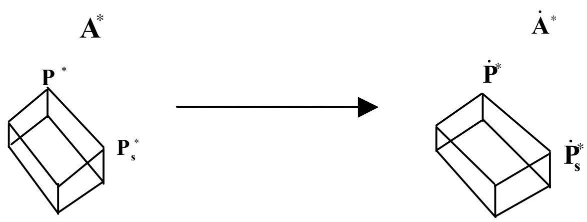


Fig. 7. Compensation of *z*-coordinates of $\mathbf{A}^*$ to obtain $\dot{\mathbf{A}}^*$.

3.4. Recovering

After the compensation above, we obtain $\dot{\mathbf{A}}^* \cong \mathbf{A}'$. To recover the original object $\mathbf{A}$ (see Fig. 4), we perform a transform $\mathbf{T_2}$ on $\dot{\mathbf{A}}^*$ where $\mathbf{T_2}$ is the inverse of $\mathbf{T_1}$ and is defined in (15). Let the recovered 3D object be $\mathbf{A}''$ and a vertex of $\mathbf{A}''$ be $\mathbf{P}''(X'', Y'', Z'')$.

$$\mathbf{P}''(X'', Y'', Z'') = \mathbf{T_2}\left(\dot{\mathbf{P}}^* \, (\dot{X}^*, \dot{Y}^*, \dot{Z}^*)\right)$$

or

$$\begin{aligned} X'' &= \dot{X}^* \cdot (Z / f) = X \\ Y'' &= \dot{Y}^* \cdot (Z / f) = Y \\ Z'' &= \dot{Z}^* \cdot (Z_s / f) \cong Z' \cdot (Z_s / f) = Z \ . \end{aligned} \tag{15}$$

It can be seen that $\mathbf{P}'' \cong \mathbf{P}$ and the recovered object $\mathbf{A}''$ is approximately equal to the original one $\mathbf{A}$.

3.5. Normalization

To compute the values of X'', Y'', Z'' in (15), we need to know Z, f and Z_s. To reduce the number of unknowns, we multiply each vertex's coordinate (X'', Y'', Z'') by a scalar $\beta = f / Z_s$ and obtain another 3D object $\mathbf{A}^{**}$ which is a normalized version of $\mathbf{A}''$ (see Fig. 8).

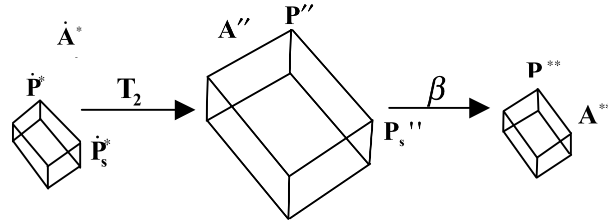


Fig. 8. Normalization.

Let the vertex of $\mathbf{A}^{**}$ be $\mathbf{P}^{**}$ (X^{**}, Y^{**}, Z^{**}). From (4), (14) and (15), we have

$$\begin{aligned} X^{**} &= X'' \cdot \beta = \dot{X}^* \cdot (Z / f) \cdot (f / Z_s) = \dot{X}^* \cdot (Z / Z_s) \\ &= \dot{X}^* \cdot (Z' / Z_s') \cong \dot{X}^* \cdot (\dot{Z}^* + f) / f \\ Y^{**} &= Y'' \cdot \beta = \dot{Y}^* \cdot (Z / f) \cdot (f / Z_s) = \dot{Y}^* \cdot (Z / Z_s) \\ &= \dot{Y}^* \cdot (Z' / Z_s') \cong \dot{Y}^* \cdot (\dot{Z}^* + f) / f \\ Z^{**} &= Z'' \cdot \beta = \dot{Z}^* \cdot (Z_s / f) \cdot (f / Z_s) = \dot{Z}^* \end{aligned} \tag{16}$$

In (16) and (13), the value of f is needed to determine X^{**}, Y^{**} and Z^{**}. The value of f here can be estimated from information provided by a user. For example, a user may identify two lines on the image, say $\overline{(X_3^{**}, Y_3^{**}, Z_3^{**})(X_4^{**}, Y_4^{**}, Z_4^{**})}$ and $\overline{(X_1^{**}, Y_1^{**}, Z_1^{**})(X_2^{**}, Y_2^{**}, Z_2^{**})}$, are of equal length. Hence we have the following equation:

$$(X_1^{**} - X_2^{**})^2 + (Y_2^{**} - Y_2^{**})^2 + (Z_1^{**} - Z_2^{**})^2 = (X_3^{**} - X_4^{**})^2 + (Y_3^{**} - Y_4^{**})^2 + (Z_3^{**} - Z_4^{**})^2 \ .$$

As f is the only unknown, so we can obtain an estimation of f by solving the equation.

3.6. A criterion for testing the proposed method

In this section, we shall examine the effectiveness of the proposed method. Objects of rectangular blocks are used in the experiments. The most noticeable distortion generated by the 3DORLD method is that parallel edges of the original object are often changed into unparallel edges on the reconstruction object.

Let us look at a block shown in Fig. 9, where three faces are visible with edges labeled. The line parallelism error θ is defined as:

$$\theta = \frac{1}{6}\sum_{i=1}^{6}\theta_i$$

where

$$\theta_1 = \cos^{-1}\left(\frac{\|l_1 \cdot l_5\|}{\|l_1\|\cdot\|l_5\|}\right), \quad \theta_2 = \cos^{-1}\left(\frac{\|l_2 \cdot l_7\|}{\|l_2\|\cdot\|l_7\|}\right),$$

$$\theta_3 = \cos^{-1}\left(\frac{\|l_5 \cdot l_6\|}{\|l_5\|\cdot\|l_6\|}\right), \quad \theta_4 = \cos^{-1}\left(\frac{\|l_3 \cdot l_7\|}{\|l_3\|\cdot\|l_7\|}\right),$$

$$\theta_5 = \cos^{-1}\left(\frac{\|l_4 \cdot l_8\|}{\|l_4\|\cdot\|l_8\|}\right) \text{ and } \theta_6 = \cos^{-1}\left(\frac{\|l_4 \cdot l_9\|}{\|l_4\|\cdot\|l_9\|}\right).$$

The ideal reconstructed 3D block should make θ close to zero.

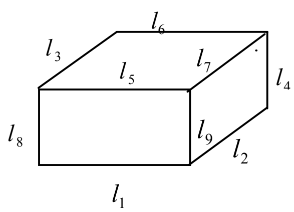
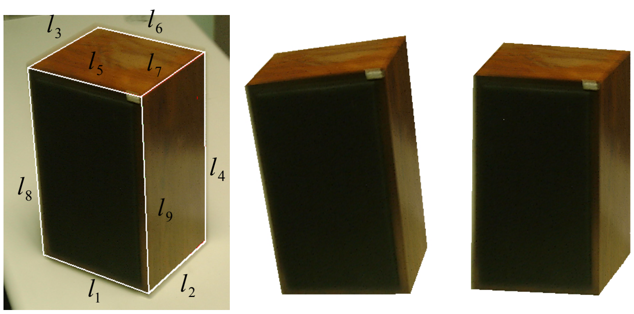


Fig. 9. A rectangular block.

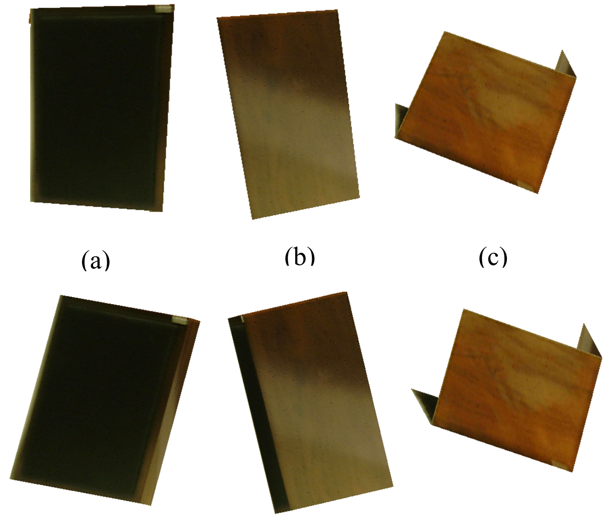
4. Experimental Results

In this section we test the proposed method in two experiments. Fig. 10(a) shows he image of a rectangular speaker with a line drawing over its edges. Fig. 10(b) shows the distorted model reconstructed by the 3DORLD method. The corrected model is illustrated in Fig. 10(c). Fig. 11 shows more views of the objects). It is apparent that the corrected object is more similar to the original box.



(a) (b) (c)

Fig. 10 (a) A rectangular speaker in the image. (b) Distorted 3D model. (c)Corrected 3D model.



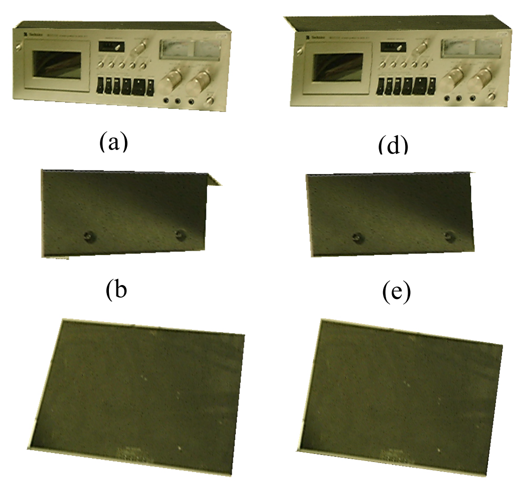
(a) (b) (c)

(d) (e) (f)

Fig.11 (a) (b) (c) Three view of the distorted model. (d) (e) (f) Three view of the corrected model corresponding to (a) (b) (c) respectively.

Fig. 12 and Fig. 13 show more examples. Again we can see that the correction of the distorted object is effective.

Fig. 12. A rectangular electronic device.



(a) (d)

(b (e)

(c) (f)

Fig. 13. (a) (b) (c) Three views of the distorted device reconstructed by the 3DORLD method. (d) (e) (f) Three views of the corresponding corrected models.

Table 1. Line parallelism error for the distorted and corrected models

Model	θ of distorted model	θ of corrected model
Speaker	5.129°	0.912°
Device	4.688°	1.175°

Table 1 shows line parallelism errors for the two 3D models. For the two corrected models, the values of the line parallelism errors are close to 0°, while those of the two distorted models are not. The results verify that the proposed method is effective to remove the distortion caused by the 3DORLD method when it is used to generate photo-realistic 3D models.

6. Conclusions

In this paper, a novel method has been proposed for the correction of the distortion on the 3D models reconstructed from images by the 3DORLD method. Experimental results show that the proposed method results in much smaller error than the original 3DORLD method.

7. Acknowledgement

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8. Reference

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